A Novel Coded Caching Scheme for Partially Cooperative Device-to-Device Networks

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***Abstract*—This paper studies coded caching in a partially cooperative D2D network, where only a subset of users transmit, while all request files. We propose a novel coded caching scheme that operates at all feasible memory regimes, regardless of the number of non-transmitting users. Compared to all known schemes, the proposed scheme requires the least amount of information about non-transmitting users, and its delivery phase is independent of the identities of non-transmitting users, thereby reducing coordination overhead. We also derive a lower bound on the transmission load. Using this, the proposed scheme is shown to be optimal in the high-memory regime.**

***Index Terms*—Coded caching, D2D network, partially cooperative, selfish users.**

I. Introduction

Cache-aided device-to-device (D2D) networks have been extensively studied due to their significance in fifth-generation (5G) and beyond cellular systems, as well as Internet of Things (IoT) networks [1]. Coded caching creates coded multicasting gains, enabling significant reductions in transmission load and improvements in transmission capacity compared to uncoded caching schemes. Coded caching in D2D networks is therefore a promising technique in modern and future systems, including 5G and beyond cellular systems, mobile edge computing, information-centric networks, and IoT networks. D2D coded caching was first studied in [2] (referred to as the JCM scheme). A D2D coded caching scheme operates in two phases: a *placement phase* followed by a *delivery phase*. In the placement phase, a central server places content in user caches, while in the delivery phase the server is absent and user demands are served through inter-user coded multicast transmissions. Most existing D2D coded caching schemes assume by default that all users transmit data during the delivery phase. However, in practical scenarios, many of the users show selfish behaviors [3]. A user who does not transmit data is referred to as a *selfish user*. The selfish behavior can be for saving energy or due to privacy and security concerns [3]. Even though selfish users do not transmit data, they can request files. A D2D network consisting of selfish users is referred to as a *partially cooperative D2D network*.

Coded caching in a partially cooperative D2D network was first studied by Tebbi and Sung in [4]. A (K, S, N) partially cooperative D2D network consists of K users, of which S are selfish, each equipped with a cache of size M files, and a central server with a library of N files. In [4], the authors proposed two schemes: a deterministic caching scheme and a random caching scheme. The scheme in [5] has an improved load compared to the deterministic caching scheme in [4]. Guan *et al.* in [6] proposed three partially cooperative D2D coded caching schemes, namely Scheme A, Scheme B, and Scheme C. Scheme A outperforms the deterministic scheme in [4] and the scheme in [5]. However, the deterministic scheme in [4], the scheme in [5] and the Scheme A in [6] operate only in the memory regime $M \geq \frac{N}{K}(S+1)$. Scheme B and Scheme C in [6] require knowledge of the identity of the selfish users before the placement phase.

In existing schemes [4]–[6], the coded transmission design explicitly depends on the identities of the non-selfish users. While such information can be used to optimize the delivery phase, it makes the transmission design sensitive to changes in the set of non-selfish users. In practice, a user expected to be a non-selfish transmitter may back out after the delivery phase has started due to constraints such as limited energy resources, unfavorable channel conditions, or local user-consent policies; in such cases, schemes that rely on non-selfish user identities require redesign or re-coordination of the coded transmissions. By contrast, an identity-agnostic transmission design remains robust, since any remaining or newly participating non-selfish user can assist in completing the delivery phase by simply transmitting the coded messages it can generate from its cache. Importantly, all previously transmitted coded messages remain useful, and no redesign of the transmissions is required.

Therefore, designing schemes that operate in all feasible memory regimes and whose placement and delivery phases are independent of selfish user identities is worth exploring. The contributions in this paper are as follows:

• A novel coded caching scheme for a partially cooperative D2D network is proposed. This scheme operates in all feasible memory regimes, regardless of the number of selfish users. Unlike existing schemes, the proposed approach does not require knowledge of the identities of selfish users to design either the placement phase or the delivery phase; it only requires knowledge of the maximum possible number of selfish users in the D2D network. For successful decoding, however, users must be aware of the set of transmitting users.

• A general cut-set based lower bound on the transmission load of a partially cooperative D2D coded caching scheme is obtained. Using this bound, the proposed scheme is shown to be optimal in the high-memory regime.

Notations: For any positive integer n and any integer $m <$

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n, $[n]$ denotes the set $\{1, 2, ..., n\}$ and $[m : n]$ denotes the set $\{m, m+1, ..., n\}$. For a set $\mathcal{A}$ and a positive integer $i \leq |\mathcal{A}|$, $\binom{\mathcal{A}}{i}$ denotes all the i-sized subsets of $\mathcal{A}$. For sets $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A}\backslash\mathcal{B}$ denotes the elements in $\mathcal{A}$ but not in $\mathcal{B}$.

II. System Model

A (K, S, N) partially cooperative D2D network consists of a central server with a library of N files $\{W_n : n \in [N]\}$ each of size B symbols over a finite field $\mathbb{F}_q$, and $K \leq N$ users $\{U_k : k \in [K]\}$ each equipped with a cache of size M files, among which S users are selfish. The condition $M \geq \frac{N}{K-S}$ is needed to ensure that any possible demands can be met using the cache contents of the $(K-S)$ non-selfish users. Coded caching scheme for such a network operates in two phases.

• **Placement phase**: During this phase, each file W_n is split into F non-overlapping subfiles (referred to as subpacketization level), i.e., $W_n = \{W_{n,i} : i \in [F]\}$. Each user U_k, where $k \in [K]$, caches MB symbols in its cache, denoted by Z_k. The cache placement can be coded or uncoded. During the placement phase, user demands are unknown. We assume that only the number of selfish users, and not their identities, is known before the placement phase.

• **Delivery phase**: During this phase, each user requests a file from $\{W_n : n \in [N]\}$, with the user demands represented by a vector $\vec{d} = (d_1, \ldots, d_K)$. Even though all users request a file, only the $(K-S)$ non-selfish users do the data transmission. Let $\mathcal{U}_{\mathcal{K}} = \bigcup_k \{U_k : k \in \mathcal{K}, \mathcal{K} \subseteq [K], |\mathcal{K}| = K - S\}$ be the set of non-selfish users. For a given $\vec{d}$, each non-selfish user $U_k \in \mathcal{U}_{\mathcal{K}}$ broadcasts a coded message $X_{k,\vec{d}}$ consisting of $S_{k,\vec{d}}$ subfiles using its cache content. The broadcast links between users are assumed to be error-free. Using these transmissions along with its cache content, each user can retrieve its requested file. The corresponding worst-case load normalized to the file size is given by $R \triangleq \max_{\vec{d}} \left(\sum_{k:U_k \in \mathcal{U}_{\mathcal{K}}} S_{k,\vec{d}}\right)/F$. The transmission of user U_k, denoted by $X_{k,\vec{d}}$, is a function only of its local cache content Z_k and the demand vector $\vec{d}$, and is independent of $\mathcal{U}_{\mathcal{K}}$. In contrast, in existing schemes the transmission design additionally depends on $\mathcal{U}_{\mathcal{K}}$.

III. Main Result

In this section, we first present a novel partially cooperative D2D coded caching scheme. Then, we derive a lower bound on the load and use it to show that the proposed scheme is optimal in a certain memory regime. Theorem 1 states the achievable load-memory pairs of the proposed scheme.

Theorem 1. *For the (K, S, N) partially cooperative D2D coded caching network, for every $t \in [0 : K-1]$, the following memory-load pair is achievable,*

$$(M, R) = \left(\frac{N(t+1)(K-1)}{(K-S)(K-1)+tS}, \frac{(K-S)(K-1)}{(K-S)(K-1)+tS} \frac{K-t-1}{t+1} \right). \quad (1)$$

Proof: The partially cooperative D2D coded caching scheme that achieves (1) is described below.

• **Placement Phase :** Let $t \in [0 : K-1]$. Each file $W_n, n \in [N]$ is divided into $(K-S)\binom{K-1}{t} + S\binom{K-2}{t-1}$ non-overlapping subfiles, i.e.,

$$W_n = \left\{ W_{n,i} : i \in \left[(K-S)\binom{K-1}{t} + S\binom{K-2}{t-1} \right] \right\}.$$

These subfiles are then encoded using a $\left[K\binom{K-1}{t}, (K-S)\binom{K-1}{t} + S\binom{K-2}{t-1}\right]$ Maximum Distance Separable (MDS) code. Recall that an $[n, k]$ MDS code has the property that the k information symbols can be reconstructed from any k out of the n coded symbols. By MDS encoding each file prior to cache placement, file recovery depends only on receiving a sufficient number of coded subfiles, rather than on the identities of specific subfiles or the users that deliver them. This property allows the delivery phase to be designed such that each non-selfish user $U_k \in \mathcal{U}_{\mathcal{K}}$ forms its coded transmissions solely based on Z_k and $\vec{d}$, without requiring knowledge of which users are selfish. Without the MDS encoding, successful decoding would rely on receiving particular subfiles, which would inherently require coordination and knowledge of the transmitting-user identities during the delivery phase. The MDS encoded subfiles of $W_n, \forall n \in [N]$, are denoted by $\left\{ Y_{n,j} : j \in \left[K\binom{K-1}{t} \right] \right\}$.

Every $j \in [K\binom{K-1}{t}]$ can be uniquely represented as $j = (k-1)\binom{K-1}{t} + \phi_k(\mathcal{T})$ for some $k \in [K]$ and $\mathcal{T} \subseteq [K]\backslash\{k\}$ with $|\mathcal{T}| = t$, where the function $\phi_k : \binom{[K]\backslash\{k\}}{t} \rightarrow \left[\binom{K-1}{t}\right]$ maps a subset $\mathcal{T}$ to its lexicographic index in $\binom{[K]\backslash\{k\}}{t}$. Then, for every $n \in [N]$ and $j \in [K\binom{K-1}{t}]$, we define $Y_{n,\mathcal{T}}^{(k)} \triangleq Y_{n,j}$, where $j = (k-1)\binom{K-1}{t} + \phi_k(\mathcal{T})$. For each user U_k, $k \in [K]$, the content stored in its cache, denoted by Z_k, is given by

$$Z_k = \left\{ Y_{n,\mathcal{T}}^{(k)} : \mathcal{T} \subseteq [K]\backslash\{k\}, |\mathcal{T}| = t, n \in [N] \right\} \cup \left(\bigcup_{\ell \in [K]\backslash\{k\}} \left\{ Y_{n,\mathcal{T}}^{(\ell)} : \mathcal{T} \ni k, \mathcal{T} \subseteq [K]\backslash\{\ell\}, |\mathcal{T}| = t, n \in [N] \right\} \right).$$

Note that, the cache size $M = |Z_k| = \{N\binom{K-1}{t}\}/\{(K-S)\binom{K-1}{t} + S\binom{K-2}{t-1}\} + \{N(K-1)\binom{K-2}{t-1}\}/\{(K-S)\binom{K-1}{t} + S\binom{K-2}{t-1}\} = \{N(t+1)(K-1)\}/\{(K-S)(K-1)+tS\}$.

• **Delivery Phase:** Suppose user U_s requests for the file W_{d_s}, where $s \in [K]$ and $d_s \in [N]$. Let $\mathcal{U}_{\mathcal{K}} = \bigcup_k \{U_k : k \in \mathcal{K}, \mathcal{K} \subseteq [K], |\mathcal{K}| = K - S\}$ be the set of transmitting users/non-selfish users. User $U_k \in \mathcal{U}_{\mathcal{K}}$ makes coded transmissions corresponding to every set $\mathcal{S} \subseteq [K]\backslash\{k\}$ with $|\mathcal{S}| = t+1$. The transmission by user U_k, $k \in \mathcal{K}$, corresponding to some $\mathcal{S} \subseteq [K]\backslash\{k\}$ with $|\mathcal{S}| = t+1$ is

$$\bigoplus_{s \in \mathcal{S}} Y_{d_s, \mathcal{S}\backslash\{s\}}^{(k)}. \quad (2)$$

Note that, user U_k has access to $Y_{n,\mathcal{T}}^{(k)}$ for every $n \in [N]$ and $\mathcal{T} \in \binom{[K]\backslash\{k\}}{t}$, and thus it can create the transmitted coded message. Consequently, the load incurred is

$$R = \frac{(K-S)\binom{K-1}{t+1}}{(K-S)\binom{K-1}{t} + S\binom{K-2}{t-1}} = \frac{(K-S)(K-1)}{(K-S)(K-1)+tS} \frac{K-t-1}{t+1}. \quad (3)$$

• **Decodability:** From the property of MDS codes, user U_k can decode its demanded file W_{d_k} from any distinct $Q = (K-S)\binom{K-1}{t} + S\binom{K-2}{t-1}$ coded subfiles of it. In other words, using any Q coded subfiles from the set $\{Y^{(\ell)}_{d_k,\mathcal{T}} : \ell \in [K], \mathcal{T} \subseteq [K]\backslash\{\ell\}, |\mathcal{T}| = t\}$, the file W_{d_k} can be decoded. Note that, for every $n \in [N]$ and $k \in [K]$, we have

$$|\left\{Y^{(k)}_{n,\mathcal{T}} : \mathcal{T} \subseteq [K]\backslash\{k\}, |\mathcal{T}| = t\right\}| = \binom{K-1}{t},$$

and for every $\ell \in [K]\backslash\{k\}$, we have

$$|\left\{Y^{(\ell)}_{n,\mathcal{T}} : \mathcal{T} \ni k, \mathcal{T} \subseteq [K]\backslash\{\ell\}, |\mathcal{T}| = t\right\}| = \binom{K-2}{t-1}.$$

Thus, each user has access to $Q_z = \binom{K-1}{t} + (K-1)\binom{K-2}{t-1}$ coded subfiles of every file from their caches. We now consider the decodability of selfish and non-selfish users separately.

First, let user U_k, $k \in \mathcal{K}$, be a non-selfish user. Let $k' \in \mathcal{K}$ such that $k' \neq k$. Consider a set $\mathcal{T} \subseteq [K]\backslash\{k, k'\}$ such that $|\mathcal{T}| = t$, and define the set $\mathcal{S} = \mathcal{T} \cup \{k\}$. Therefore, we have $|\mathcal{S}| = t+1$. Consider the following transmission from user $U_{k'}$, corresponding to the set $\mathcal{S}$

$$\bigoplus_{s\in\mathcal{S}} Y^{(k')}_{d_s,\mathcal{S}\backslash\{s\}} = Y^{(k')}_{d_k,\mathcal{T}} \oplus \left(\bigoplus_{s\in\mathcal{S}, s\neq k} Y^{(k')}_{d_s,\mathcal{S}\backslash\{s\}}\right). \quad (4)$$

Since $k \in \mathcal{S}\backslash\{s\}$ for every $s \neq k$, user U_k has the coded subfile $Y^{(k')}_{d_s,\mathcal{S}\backslash\{s\}}$, for every $k' \in \mathcal{K}$, in its cache. Therefore, from the delivery phase, user U_k can decode $Y^{(k')}_{d_k,\mathcal{T}}$ for every $k' \in \mathcal{K}\backslash\{k\}$ and $\mathcal{T} \subseteq [K]\backslash\{k, k'\}$. For a $k, k' \in \mathcal{K}$, we have

$$\left|\left\{Y^{(k')}_{d_k,\mathcal{T}} : \mathcal{T} \subseteq [K]\backslash\{k, k'\}\right\}\right| = \binom{K-2}{t}.$$

Thus, in addition to the subfiles in the cache, user U_k can get $Q_T = (K-S-1)\binom{K-2}{t-1}$ coded subfiles of W_{d_k} from the delivery phase. Therefore, user U_k has a total of $Q_z + Q_T$

$$= \binom{K-1}{t} + (K-1)\binom{K-2}{t-1} + (K-S-1)\binom{K-2}{t}$$
$$= (K-S)\binom{K-1}{t} + S\binom{K-2}{t-1} = Q$$

coded subfiles of W_{d_k}. Thus, the user U_k can decode W_{d_k}.

Let user U_k, $k \in [K]\backslash\mathcal{K}$, be a selfish user. Consider user $U_{k'}$ such that $k' \in \mathcal{K}$. Consider a set $\mathcal{T} \subseteq [K]\backslash\{k, k'\}$ such that $|\mathcal{T}| = t$, and define the set $\mathcal{S} = \mathcal{T} \cup \{k\}$. Therefore, we have $|\mathcal{S}| = t+1$. Consider the following transmission from user $U_{k'}$, corresponding to the set $\mathcal{S}$:

$$\bigoplus_{s\in\mathcal{S}} Y^{(k')}_{d_s,\mathcal{S}\backslash\{s\}} = Y^{(k')}_{d_k,\mathcal{T}} \oplus \left(\bigoplus_{s\in\mathcal{S}, s\neq k} Y^{(k')}_{d_s,\mathcal{S}\backslash\{s\}}\right). \quad (5)$$

Since $k \in \mathcal{S}\backslash\{s\}$ for every $s \neq k$, user U_k has the coded subfile $Y^{(k')}_{d_s,\mathcal{S}\backslash\{s\}}$, for every $k' \in \mathcal{K}$, in its cache. Therefore, from the delivery phase, user U_k can decode $Y^{(k')}_{d_k,\mathcal{T}}$ for every $k' \in \mathcal{K}$ and $\mathcal{T} \subseteq [K]\backslash\{k, k'\}$. For a $k \in [K]\backslash\mathcal{K}$ and $k' \in \mathcal{K}$,

we have $\left|\left\{Y^{(k')}_{d_k,\mathcal{T}} : \mathcal{T} \subseteq [K]\backslash\{k, k'\}\right\}\right| = \binom{K-2}{t}.$

Thus, in addition to the subfiles in the cache, user U_k can get $Q_T = (K-S)\binom{K-2}{t-1}$ coded subfiles of W_{d_k} from the delivery phase. Therefore, user U_k has a total of $Q_z + Q_T$

$$= \binom{K-1}{t} + (K-1)\binom{K-2}{t-1} + (K-S)\binom{K-2}{t}$$
$$> \binom{K-1}{t} + (K-1)\binom{K-2}{t-1} + (K-S-1)\binom{K-2}{t}$$

$= Q$ coded subfiles of W_{d_k}. That is, a selfish user obtains more subfiles than required to decode its demanded file. Thus, the user U_k can decode W_{d_k}. This completes the proof. ■

The proposed scheme works for any (K, S', N) network with $S' \leq S$, achieving the same load, since $(K-S') \geq (K-S)$ non-selfish users are available and the scheme can proceed by selecting any $(K-S)$ users to perform the transmissions. The following example illustrates Theorem 1.

Example 1. *Consider a $(6, 2, 6)$ network. Assume that $t = 2$. Each file is divided into 48 non-overlapping subfiles, i.e., $W_n = \{W_{n,i} : i \in [48]\}, \forall n \in [6]$. These subfiles are then encoded using a $[60, 48]$ MDS code. The coded subfiles of each file W_n is represented as $[Y_{n,j} : j \in [60]]$.*

Every $j \in [60]$ can be uniquely represented as $j = (k-1)10 + \phi_k(\mathcal{T})$ for some $k \in [6]$ and $\mathcal{T} \subseteq [6]\backslash\{k\}$ with $|\mathcal{T}| = 2$, where the function $\phi_k : \binom{[6]\backslash\{k\}}{2} \to [10]$ maps a subset $\mathcal{T}$ to its lexicographic index in $\binom{[6]\backslash\{k\}}{2}$.

The cache content of user U_k, denoted by Z_k, is given by

$$Z_k = \left\{Y^{(k)}_{n,\mathcal{T}} : \mathcal{T} \subseteq [6]\backslash\{k\}, |\mathcal{T}| = 2, n \in [6]\right\} \cup$$
$$\left(\bigcup_{\ell\in[6]\backslash\{k\}} \left\{Y^{(\ell)}_{n,\mathcal{T}} : \mathcal{T} \ni k, \mathcal{T} \subseteq [6]\backslash\{\ell\}, |\mathcal{T}| = 2, n \in [6]\right\}\right).$$

Note that there are $\binom{5}{2} + 5 \times 4 = 30$ coded subfiles of each of 6 files in each user's cache, and each coded subfile is of 1/48 file size. Therefore, $M = 6 \times 30 \times \frac{1}{48} = \frac{15}{4}$ files. Let $\vec{d} = (1, 2, 3, 4, 5, 6)$, and let U_4 and U_5 be the selfish users. Thus the set of transmitting users is $\mathcal{U}_{\{1,2,3,6\}}$. User U_k, $k \in \{1, 2, 3, 6\}$ makes coded transmissions corresponding to every set $\mathcal{S} \in \binom{[6]\backslash\{k\}}{3}$. Each of the four transmitting users thus transmits 10 coded subfiles. Therefore, $R = 4 \times 10 \times \frac{1}{48} = \frac{5}{6}$.

Now, consider user U_1. User U_1 already has 30 coded subfiles of the demanded file W_1 in its cache. It requires 18 more coded subfiles of W_1 to decode W_1, since we have used a $[60, 48]$ MDS code to encode the subfiles. Consider a user from $\mathcal{U}_{\{1,2,3,6\}}$ other than U_1, say U_2. Consider a set $\mathcal{T} \subseteq [6]\backslash\{1, 2\}$ such that $|\mathcal{T}| = 2$, i.e., $\mathcal{T} \in \{\{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}, \{5, 6\}\}$. Corresponding to each $\mathcal{T}$, define the set $\mathcal{S} = \mathcal{T} \cup \{1\}$, i.e., $\mathcal{S} \in \{\{1, 3, 4\}, \{1, 3, 5\}, \{1, 3, 6\}, \{1, 4, 5\}, \{1, 4, 6\}, \{1, 5, 6\}\}$. Corresponding to $\mathcal{S} = \{1, 3, 4\}$, user U_2 transmits $Y^{(2)}_{d_1,\{3,4\}} \oplus Y^{(2)}_{d_3,\{1,4\}} \oplus Y^{(2)}_{d_4,\{1,3\}} = Y^{(2)}_{1,\{3,4\}} \oplus Y^{(2)}_{3,\{1,4\}} \oplus Y^{(2)}_{4,\{1,3\}}$. From this transmission, user U_1 will get the coded subfiles $Y^{(2)}_{1,\{3,4\}}$ of the demanded file W_1 since it have the subfiles $Y^{(2)}_{3,\{1,4\}}$ and $Y^{(2)}_{4,\{1,3\}}$ in its cache. Similarly, from the transmissions by user U_2 corresponding to $\mathcal{S} = \{1, 3, 5\}, \{1, 3, 6\}, \{1, 4, 5\}, \{1, 4, 6\}$ and $\{1, 5, 6\}$, *user U_1 will get the coded subfile $Y^{(2)}_{1,\{3,5\}}, Y^{(2)}_{1,\{3,6\}}, Y^{(2)}_{1,\{4,5\}}, Y^{(2)}_{1,\{4,6\}}$ and $Y^{(2)}_{1,\{5,6\}}$, respectively. That is, U_1 gets 6 coded subfiles of the demanded file W_1 from the transmissions of U_2, and similarly six coded subfiles from each of U_3 and U_6. Thus, U_1 obtains a total of 18 coded subfiles of W_1 from the transmissions, and hence can decode W_1.*

Next, we derive a lower bound on the load using the cut-set argument. This bound is obtained under the assumption of a symmetric load, i.e., the transmissions made by each non-selfish user are of equal size. This assumption was previously used in [7], where a lower bound on the optimal load–memory trade-off was derived for the D2D coded caching problem in which selfish users are not present. Further, this assumption is relevant to our scenario since the delivery design must be identity-agnostic to unknown realizations of non-selfish users, necessitating a symmetric per-user transmission load.

Theorem 2. *For the $(K, S, N \geq K)$ partially cooperative D2D coded caching scheme, the optimal-load memory trade-off is lower bounded by*

$$R^*(M) \geq \max_{s\in[K-1]} \frac{N - sM}{\frac{\max(K-S-s,1)}{K-S}\lceil N/s\rceil}. \quad (6)$$

Proof: Let $s \in [K-1]$. Consider s users in the network such that the set of remaining $K-s$ users includes at least one non-selfish user. Among the considered s users, assume that $\ell \leq \min(s, S)$ of them are selfish users. Since the set of s users can include at most $K-S-1$ users, we have $\ell \geq \max(0, s-(K-S)+1)$. Without loss of generality, we assume that the chosen s users are indexed from 1 to s. Now, consider the demand vector $\vec{d_1} = (1, 2, \ldots, s, \emptyset, \ldots, \emptyset)$, where user U_k demands for the file W_k for every $k \in [s]$. The demands of the rest of the users are arbitrary. Let $\mathcal{U}$ be the set of non-selfish users not part of the set of considered s users. Note that $|\mathcal{U}| = (K-S)-(s-\ell) \geq 1$. Let $\tilde{X}_1$ denotes the transmissions from users in $\mathcal{U}$ corresponding to the demand vector $\vec{d_1}$. Now, consider another demand vector $\vec{d_2} = (s{+}1, s{+}2, \ldots, 2s, \emptyset, \ldots, \emptyset)$ and denote the transmission from users in $\mathcal{U}$ corresponding to $\vec{d_2}$ as $\tilde{X}_2$. Similarly, we consider $\lceil N/s\rceil$ such demand vectors to cover the entire file library and the corresponding transmissions. Then, we get

$$N = H(W_{[1:N]}) \leq H(Z_{[1:s]}, \tilde{X}_{[1:\lceil N/s\rceil]}) \quad (7a)$$

$$\leq H(Z_{[1:s]}) + H(\tilde{X}_{[1:\lceil N/s\rceil]}) \quad (7b)$$

$$\leq sM + \frac{(K-S)-(s-\ell)}{K-S}\left\lceil \frac{N}{s}\right\rceil R^*(M), \quad (7c)$$

where (7a) follows from the fact that the entire file library can be decoded using the cache contents of the chosen s users and transmissions $\tilde{X}_{[1:\lceil N/s\rceil]}$ from users in $\mathcal{U}$. In addition, (7c) follows from our symmetric load assumption, where the size of the transmissions from users in $\mathcal{U}$ is $|\mathcal{U}|/(K-S)$ fraction of the load. Rearranging the final inequality, for every choices of $s \in [K-1]$ and $\ell \in [\max(0, s-(K-S)+1) : \min(s, S)]$, we have

$$R^*(M) \geq \frac{N - sM}{\frac{(K-S)-(s-\ell)}{K-S}\left\lceil \frac{N}{s}\right\rceil}. \quad (8)$$

Since the RHS of the inequality decreases with increasing ℓ, in order to maximize the lower bound, we choose $\ell = \max(0, s-(K-S)+1)$. Therefore, we get

$$R^*(M) \geq \max_{s\in[K-1]} \frac{N - sM}{\frac{(K-S)-(s-\max(0,s-(K-S)+1))}{K-S}\lceil N/s\rceil}$$

$$= \max_{s\in[K-1]} \frac{N - sM}{\frac{\max(K-S-s,1)}{K-S}\lceil N/s\rceil}. \quad \blacksquare$$

Next, we show the optimality of the proposed scheme in a certain memory regime.

Theorem 3. *The proposed scheme is optimal when $\frac{1}{1+\frac{K-S-1}{(K-1)^2}} \leq \frac{M}{N} \leq 1$ and the corresponding optimal load is*

$$R^*(M) = (K-S)\left(1 - M/N\right)/(K-S-1). \quad (9)$$

Proof: By substituting, $s = 1$ in (6), we have the following lower bound on the optimal memory-load trade-off

$$R^*(M) \geq (K-S)\left(1 - M/N\right)/(K-S-1). \quad (10)$$

By substituting $t = K-2$ in (1), we get $M_1 = N/\{1 + \frac{K-S-1}{(K-1)^2}\}$ and the corresponding load achieved is $R_1 = \frac{K-S}{K(K-1)-S}$. Similarly, corresponding to $t = K-1$, the memory-load pair $(M_2, R_2) = (N, 0)$ is achievable. By memory sharing, the load $R(M) = \frac{K-S}{K-S-1}\left(1 - \frac{M}{N}\right)$ is achievable for every $1/\{1 + \frac{K-S-1}{(K-1)^2}\} \leq M/N \leq 1$. $\blacksquare$

TABLE I: Comparison of all known and the proposed (K, S, N) partially feasible cooperative D2D coded caching schemes.

Sl. No.	Schemes and Parameters	Information level (selfish users)	Cache Placement	Delivery Scheme	Memory M	Operating memory regime	Transmission load $R(M)$
1	Deterministic caching scheme in [4], $t \in \mathbb{Z}^+$	Level 2	Centralized and Uncoded Placement	Dependent on selfish-user identities	$\frac{tN}{K}$	$\frac{N}{K}(S+1) \leq M \leq N$	$\frac{1}{t\binom{K}{t}}\sum_{i=0}^{S}\binom{S}{i}\binom{K-S}{t+1-i}\left(t+1+\left\lceil\frac{i}{t-i}\right\rceil\right)$
2	Random caching scheme in [4], $M \in [N/(K-S) : N]$	Level 3	Decentralized and Coded Placement	Dependent on selfish-user identities	$[\frac{N}{K-S} : N]$	All feasible memory regime $\frac{N}{K-S} \leq M \leq N$	$\frac{1}{r}\sum_{i=2}^{K} R(i)\left(\frac{Mr}{N}\right)^{i-1}\left(1-\frac{Mr}{N}\right)^{K-i+1}$, refer [4] for r and $R(i)$ expressions.
3	Scheme in [5], $t \in \mathbb{Z}^+$	Level 2	Centralized and Uncoded Placement	Dependent on selfish-user identities	$\frac{tN}{K}$	$\frac{N}{K}(S+1) \leq M \leq N$	$\frac{1}{t\binom{K}{t}}\sum_{i=0}^{S}\left[\binom{S}{i}\binom{K-S}{t+1-i}(t+1)+\phi_i\right]$, where $\phi_i = \min\left(i\left\lceil\frac{S}{t-i}\right\rceil\binom{K-S}{t+1-i}, \binom{S}{i}\left\lceil\frac{i}{t-i}\right\rceil\binom{K-S}{t+1-i}\right)$
4	Scheme A in [6] $t \in [K]$	Level 2	Centralized and Uncoded Placement	Dependent on selfish-user identities	$\frac{tN}{K}$	$\frac{N}{K}(S+1) \leq M \leq N$	$\frac{1}{\binom{K}{t}}\sum_{i=0}^{S}\binom{S}{i}\binom{K-S}{t+1-i}\frac{t+1-i}{t-i}$
5	Scheme B in [6], $S = 1, q \in [K-1]$	Level 4	Centralized and Uncoded Placement	Dependent on selfish-user identities	$N\frac{q(K-q-1)+1}{K(K-q-1)+1}$	$\frac{N}{K}(S+1) \leq M \leq N$	$(K/q)[1 - M/N]$
6	Scheme C in [6], $t_S \in [S], t_A = \frac{(K-S)t_S}{S}$	Level 4	Centralized and Uncoded Placement	Dependent on selfish-user identities	$\frac{t_S N}{S}$	$N/[\min(S, K-S)] \leq M \leq N$	$(N/M - 1) + ([S\left(1 - M/N\right)]/[1 + SM/N])$
7	Proposed scheme in Theorem 1, $t \in [0 : K-1]$	Level 1	Centralized and Coded Placement	Independent of selfish-user identities	$\frac{N(t+1)(K-1)}{(K-S)(K-1)+tS}$	All feasible memory regime $N/(K-S) \leq M \leq N$	$\frac{(K-S)(K-1)}{(K-S)(K-1)+tS}\frac{K-t-1}{t+1}$

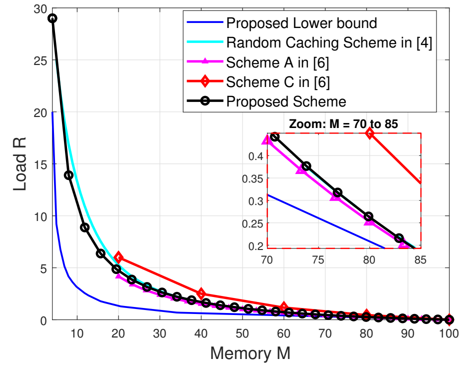
IV. Performance Analysis

In this section, performance analysis of the proposed scheme is carried out by comparing with all known schemes. Different partially cooperative D2D coded caching schemes vary in the amount of information required about selfish users. To categorize the schemes, we define different levels of information, listed below from the least to the most required:

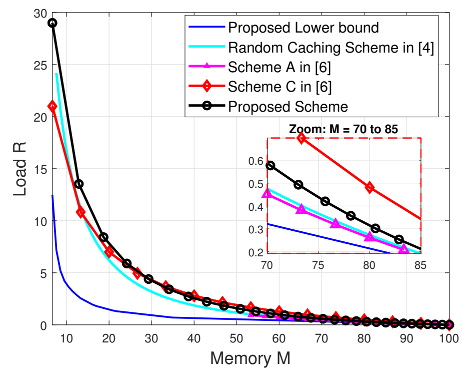
- Level 1: Knowledge of the maximum number of selfish users before the placement phase,
- Level 2: No knowledge of selfish users before the placement phase, but knowledge of their identities to design the delivery messages,
- Level 3: Knowledge of the maximum number of selfish users before the placement phase and knowledge of their identities to design the delivery messages,
- Level 4: Knowledge of the identities of selfish users before the placement phase.

Table I summarizes the level of information required and the operating memory regimes of all schemes. Among them, the proposed scheme requires the least information about selfish users. Unlike existing schemes, its transmission design does not depend on the identities of selfish users. Among all the centralized schemes, only the proposed scheme operates in all feasible memory regimes, regardless of the number of selfish users. This follows from the fundamental limitation of uncoded placement, which requires the file library to be duplicated at least $S+1$ times across the user caches to tolerate S selfish users, yielding $KM \geq (S+1)N$, i.e., $M \geq (S+1)N/K$.

Fig.1 compares the proposed scheme with existing schemes for a $(30, S, 100)$ network for different values of S. Among the schemes requiring level 2 information about selfish users and operating in the regime $M \geq \frac{N}{K}(S+1)$—namely the deterministic scheme in [4], the scheme in [5] and the Scheme A in [6], Scheme A in [6] performs best and is therefore the only one plotted. The figure highlights memory regimes where only the proposed scheme operates. The Scheme B in [6] was studied only for $S = 1$. Compared to Scheme C in [6], the proposed scheme, despite requiring less selfish-user information, achieves a lower load over the entire memory regime for small S and in the high-memory regime for all S. Compared to the random caching scheme in [4], the proposed scheme achieves a lower load in the lower memory regime and both scheme approaches the same load at higher regimes, when the number of selfish users S is small. As S increases, the load reduction achieved by the proposed scheme gradually diminishes and, for sufficiently large S, the proposed scheme incurs a higher load than the existing schemes, particularly in the low-memory regime. This behavior is a consequence of the *user-identity-agnostic* coded transmission design adopted in the delivery phase of our scheme.



(a) $S = 5$



(b) $S = 15$

Fig. 1: Load–memory trade off of the proposed and existing schemes for a $(30, S, 100)$ partially cooperative D2D network.

V. Conclusion

In this work, we first proposed a novel partially cooperative D2D coded caching scheme that operates at all feasible memory regimes. Unlike existing schemes, the proposed scheme does not require knowledge of the selfish user identities; it only needs to know the maximum possible number of selfish users in the network. A general cut-set based lower bound on the load is also obtained. Using this, the proposed scheme is shown to be optimal in the high-memory regime. Obtaining tighter converse bounds and improved optimality guarantees are directions for future work. Designing schemes that further reduce the load by adapting to the exact number of selfish users is also an interesting research direction.

References

[1] D. Prerna, T. Rajkumar, and Neeraj Kumar, "Device-to-device content caching techniques in 5G: A taxonomy, solutions, and challenges," *Computer Communications*, vol. 153, pp. 48-84, March 2020.

[2] M. Ji, G. Caire, and A. F. Molisch, "Fundamental Limits of Caching in Wireless D2D Networks," *IEEE Trans. Inf. Theory*, vol. 62, no. 2, pp. 849-869, Feb. 2016.

[3] C. Gao, H. Zhang, X. Chen, Y. Li, D. Jin, and S. Chen, "Impact of Selfishness in Device-to-Device Communication Underlying Cellular Networks," *IEEE Trans. Veh. Technol.*, vol. 66, no. 10, pp. 9338-9349, Oct. 2017.

[4] A. Tebbi and C. W. Sung, "Coded caching in partially cooperative D2D communication networks," in *Int. Congr. Ultra Modern Telecommun. Control Syst. Workshops (ICUMT)*, Munich, Germany, 2017, pp. 148-153.

[5] A. Phatak and M. K. Varanasi, "An improved coded caching scheme for partially cooperative D2D networks," in *IEEE Int. Symp. Inf. Theory (ISIT)*, Espoo, Finland, 2022, pp. 1121-1126.

[6] W. Guan, K. Huang, X. Xie, J. Zhang, K. Cai, and X. Huang, "New Results on Coded Caching in Partially Cooperative D2D Networks," in *IEEE Int. Symp. Inf. Theory (ISIT)*, Athens, Greece, 2024, pp. 1556-1561.

[7] A. Sengupta and R. Tandon, "Beyond cut-set bounds - the approximate capacity of D2D networks," in *Inf. Theory Appl. Workshop (ITA)*, San Diego, CA, USA, 2015, pp. 78-83.