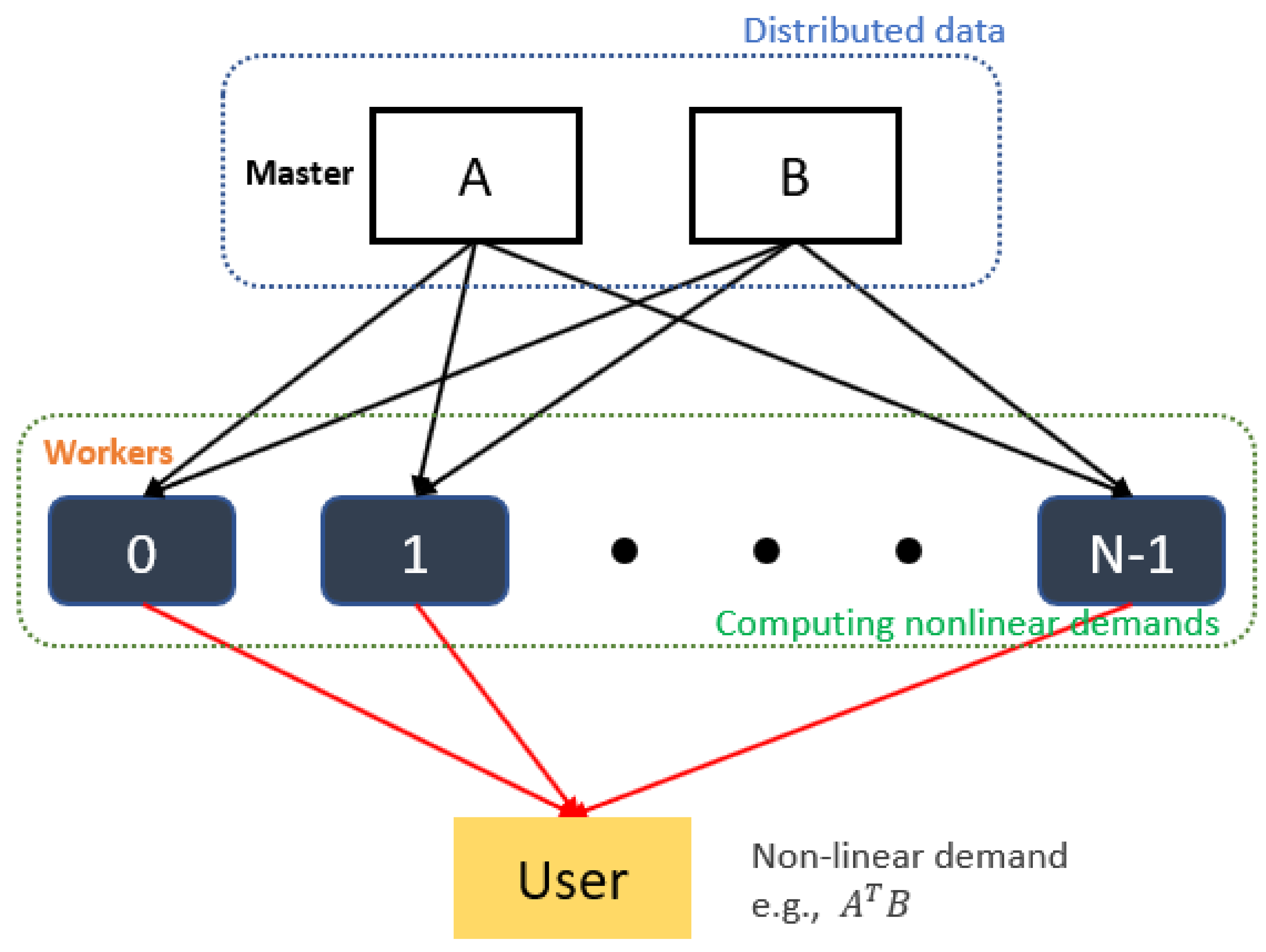
Structured Polynomial Codes

Ahmad Tanha, Mohammad Reza Deylam-Salehi, Derya Malak
Communication Systems Department, EURECOM
Biot, Sophia Antipolis, France
{tanha, deylam, malak}@eurecom.fr

Motivation



A fundamental challenge in distributed computing systems:

Balancing **computation** and **communication** complexity

Related work

Distributed computing frameworks:

- MapReduce [1], Hadoop, Spark [2], TeraSort [3]

Channel coding approaches:

- Polynomial codes, Lagrange coded computing [4, 5]

Source coding approaches:

- Structured codes for modulo two sum computation in [6], and distributed matrix multiplication in [7]

Contributions

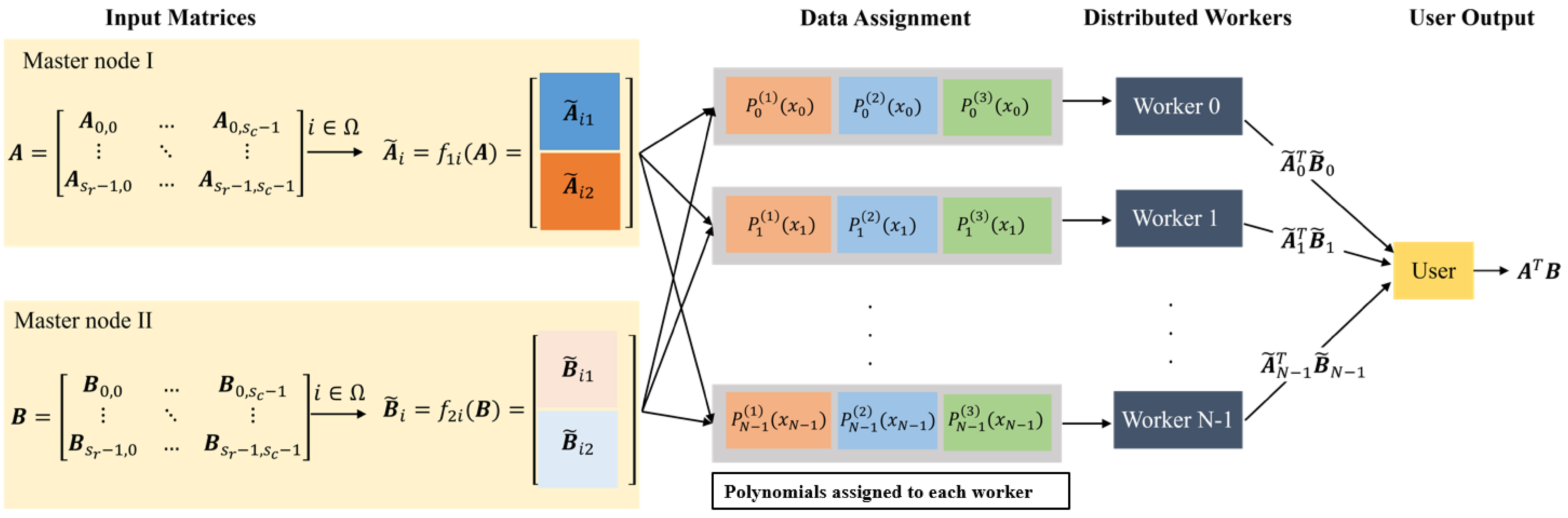
Novelty:

- Combining the benefits of structured coding and polynomial codes
- Elevating the Körner-Marton approach to the distributed matrix multiplication setting
- Incorporating a secure matrix multiplication design

Savings:

- Low complexity distributed encoding
- Communication costs (reduced by %50)
- Storage size (reduced by %50)

A structured distributed matrix multiplication model



- ☐ *Each worker, using the assigned polynomials, calculates the product of sub-matrices* $\tilde{\mathbf{A}}_i^\intercal \tilde{\mathbf{B}}_i$.
- ☐ *Using* $\{\tilde{\mathbf{A}}_i^\intercal \tilde{\mathbf{B}}_i\}_i$ *from a subset of workers, the user decodes* $\mathbf{AB}$.
- ☐ *The user cannot decode* $\mathbf{A}$ *or* $\mathbf{B}$, *where the security of matrix multiplication is ensured by structured coding.*

Source coding for matrix multiplication [7]

Two *distributed sources*, $\mathbf{A} \in \mathbb{F}_q^{m\times 1}$ and $\mathbf{B} \in \mathbb{F}_q^{m\times 1}$:

- Splitting of each source:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}^\intercal \in \mathbb{F}_q^{m\times 1} , \qquad \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \in \mathbb{F}_q^{m\times 1} ,$$

- Nonlinear mapping from each source:

$$\mathbf{X}_1 = g_1(\mathbf{A}) = \begin{bmatrix} \mathbf{A}_2 \\ \mathbf{A}_1 \\ \mathbf{A}_2^\intercal \mathbf{A}_1 \end{bmatrix} \in \mathbb{F}_2^{(m+1)\times 1} , \qquad \mathbf{X}_2 = g_2(\mathbf{B}) = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \mathbf{B}_1^\intercal \mathbf{B}_2 \end{bmatrix} \in \mathbb{F}_2^{(m+1)\times 1} .$$

- Linear encoding: Sources use a common encoder, compute and send $\mathbf{C}\mathbf{X}_j^n \in \mathbb{F}_2^{(m+1)\times k}$ [6].
- Decoding: Exploiting [6], the sum rate needed for the user to recover the vector sequence

$$\mathbf{Z}^n = \mathbf{X}_1^n \oplus_2 \mathbf{X}_2^n \in \mathbb{F}_2^{(m+1)\times n}$$

with a vanishing error probability, is determined as:

$$R_{\mathrm{KM}}^{\Sigma} = 2H(\mathbf{X}_1 \oplus_2 \mathbf{X}_2) = 2H(\mathbf{U}, \mathbf{V}, \mathbf{W}) ,$$

where the following vectors can be computed in a fully distributed manner:

$$\mathbf{U} = A_2 \oplus_q B_1 \in \mathbb{F}_q^{m/2\times 1} , \quad \mathbf{V} = A_1 \oplus_q B_2 \in \mathbb{F}_q^{m/2\times 1} , \quad \mathbf{W} = A_2^T A_1 \oplus_q B_1^T B_2 \in \mathbb{F}_q .$$

The user can recover the desired inner product using $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{W}$.

Future directions

Structured codes for

- n-matrix products
- privacy/security aspects
- tensor product computations

References

[1] Jeffrey Dean and Sanjay Ghemawat. MapReduce: simplified data processing on large clusters. *Communications of the ACM*, 51(1):107–113, 2008.

[2] Matei Zaharia, Mosharaf Chowdhury, Michael J Franklin, Scott Shenker, and Ion Stoica. Spark: Cluster computing with working sets. In *2nd USENIX Workshop on Hot Topics in Cloud Computing (HotCloud 10)*, 2010.

[3] Alkatheri et al. A comparative study of big data frameworks. *Int. Jour. Comp. Sci. IJCSIS*, 2019.

[4] López et al. Secure MatDot codes: a secure, distributed matrix multiplication scheme. In *ITW 2022*, Mumbai, India, 2022.

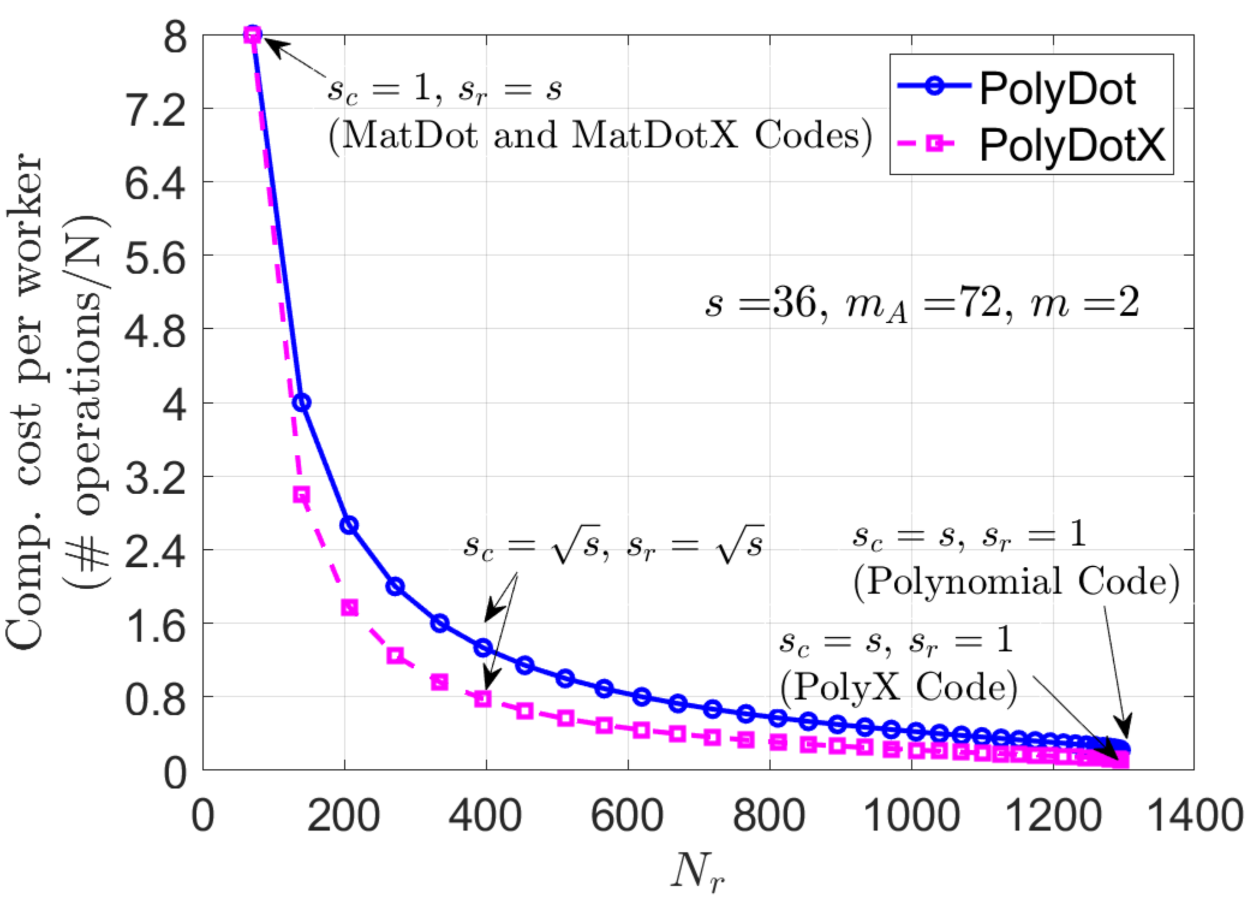
[5] Yu et al. Lagrange coded computing: Optimal design for resiliency, security, and privacy. In *Proc. Int. on AI and Stat.*, 2019.

[6] Körner and Marton. How to encode the modulo-two sum of binary sources. *IEEE Trans. Inf. Theory*, 1979.

[7] Malak. Distributed structured matrix multiplication. In *ISIT*, Athens, Greece, Jul. 2024.

Performance results

For $s_c \gg m$, the upper bound of computation cost per worker approaches $1 + \frac{1}{2s}$.



The total communication cost is reduced by %50 compared to the PolyDot model.