Blind Channel Identification Based On Cyclic Statistics.

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Abstract : Blind channel identification and equalization based on second-order statistics by subspace fitting and linear prediction have received a lot of attention lately. On the other hand, the use of cyclic statistics in fractionally sampled channels has also raised considerable interest. We propose to use these statistics in subspace fitting and linear prediction for (possibly multiuser and multiple antennas) channel identification. We base our identification schemes on the cyclic statistics, using the stationary multivariate representation introduced by [6] and [9] [10]. This leads to the use of all cyclic statistics. The methods proposed, compared to classic approaches, have equivalent performance for the subspace fitting and enhanced performance for linear prediction.

1 Introduction

Major impairments of most wireless communication channels, especially in mobile environments, are intersymbol interference (ISI), cochannel interference (CCI) and adjacent channel interference (ACI). In wireless networks, the latter is solved by source separation techniques and ISI by equalization techniques. In the past three decades, so-called "blind" channel identification and equalization techniques flourished ; where "blind" really means based on the outputs of the channel only; and some assumptions on the nature of the input and/or channel. Among these techniques, methods based on second-order statistics only are very attractive, because they need few samples to allow channel identification compared to the other methods (implicitly or explicitly based on higher order statistics).

Recognizing that communication (continuous time) signals are cyclostationary shows the cyclostationarity of the oversampled (w.r.t. the baud rate) discrete time signals and, under mild conditions, leads to the identifiability of the channel. The optimal methodis the covariance matching, introduced by [5]. The two other families of methods are subspace fitting and linear prediction introduced with non-cyclic statistics [14] which are suboptimal, but do not need complex numerical searches as the covariance matching method.

In this paper, we introduce a new multichannel channel model derived from the stationary multivariate representation introduced by [6]. This representation allows us to derive the subspace fitting and linear prediction methods using the cyclic statistics. Algebraic considerations show that the cyclic subspace fitting has, in theory, the same performance as the non-cyclic subspace fitting, although the cyclic approach is characterized by fewer parameters for the channel, leading to some enhancement w.r.t. the non-cyclic method. For the linear prediction, basing the prediction on more samples leads to better performance.

2 Data Model

We consider a spatial division multiple access (S.D.M.A.) communication system with p emitters and a receiver constituted of an array of M antennas. The signals received are oversampled by a factor m w.r.t. the symbol rate. The channel is FIR of duration NT/m where T is the symbol duration.

The received signal can be written as :

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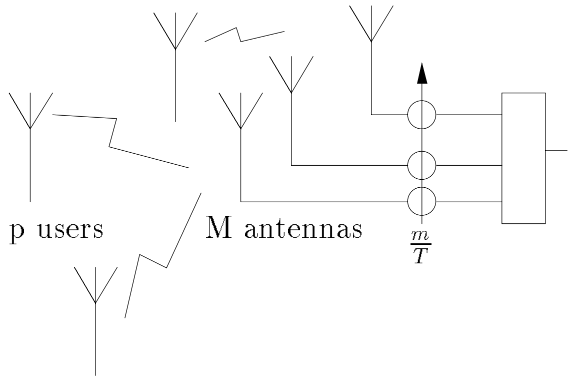

Figure 1: Schematic S.D.M.A. situation

$$\mathbf{x}(n) = \sum_{k=0}^{N-1} \mathbf{h}(k)\mathbf{u}(n \Leftrightarrow k) + \mathbf{v}(n) = \sum_{k=\lceil \frac{n}{m} \rceil}^{\lfloor \frac{n+N-1}{m} \rfloor} \mathbf{h}(n \Leftrightarrow km)\mathbf{a}_k + \mathbf{v}(n) \tag{1}$$

where $\mathbf{u}(n) = \sum_{k=-\infty}^{\infty} \mathbf{a}_k \delta(n \Leftrightarrow km)$

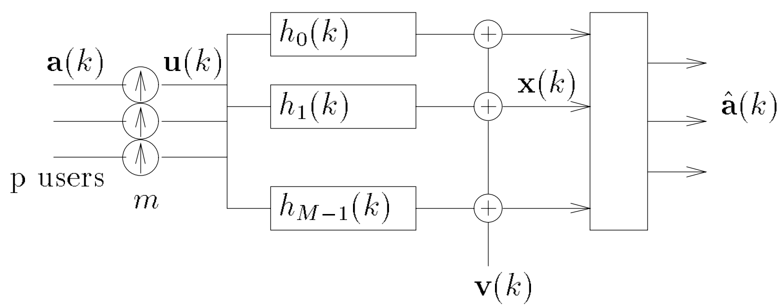

Figure 2: Channel model

The received signal $\mathbf{x}(n)$ and noise $\mathbf{v}(n)$ are $M \times 1$ vectors. $\mathbf{x}(n)$ is cyclostationary with period m whereas $\mathbf{v}(n)$ is assumed not to be cyclostationary with period m. $\mathbf{h}(k) = [\mathbf{h}_1(k)^T \cdots \mathbf{h}_M(k)^T]^T$ has dimension $M \times p$, $\mathbf{a}(k)$ and $\mathbf{u}(k)$ have dimensions $p \times 1$.

3 Cyclic Statistics

Following the assumptions here above, the correlations :

$$\mathbf{R}_{xx}(n, \tau) = \mathrm{E}\left\{\mathbf{x}(n)\mathbf{x}^H(n \Leftrightarrow \tau)\right\} \tag{2}$$

are cyclic in n with period m (^{H} denotes complex conjugate transpose) [4]. One can easily express them as:

$$\mathbf{R}_{xx}(n, \tau) = \sum_{\alpha=-\infty}^{\infty} \sum_{\beta=-\infty}^{\infty} \mathbf{h}(n \Leftrightarrow \alpha m)\mathbf{R}_{aa}(\beta)\mathbf{h}^H(n \Leftrightarrow \alpha m + \beta m \Leftrightarrow \tau) + \mathbf{R}_{vv}(\tau) \tag{3}$$

We then express the k^{th} cyclic correlation as :

$$\mathbf{R}_{xx}^{\{k\}}(\tau) \triangleq \frac{1}{m} \sum_{l=0}^{m-1} \mathbf{R}_{xx}(l, \tau) e^{-j\frac{2\pi l k}{m}} = \mathrm{E}^k \left\{\mathbf{x}(l)\mathbf{x}^H(l \Leftrightarrow \tau)\right\} \tag{4}$$

whose value is :

$$\mathbf{R}_{xx}^{\{k\}}(\tau) = \frac{1}{m} \sum_{\alpha=-\infty}^{\infty} \sum_{\beta=-\infty}^{\infty} \mathbf{h}(\alpha)\mathbf{R}_{aa}(\beta)\mathbf{h}^{H}(\alpha + \beta m \Leftrightarrow \tau)e^{-\jmath\frac{2\pi\alpha k}{m}} + \mathbf{R}_{vv}(\tau)\delta(k) \tag{5}$$

with k integer and $\mathbf{R}_{xx}^{\{k\}}(\tau) = \mathbf{R}_{xx}^{\{k+m\}}(\tau)$ (usually, one uses $k \in [\Leftrightarrow\left\lfloor \frac{m}{2} \right\rfloor, \cdots \left\lfloor \frac{m}{2} \right\rfloor]$).

Let's also denote $\mathcal{T}_K(\mathbf{H}_N)$ as the convolution matrix of $\mathbf{H}_N = [\mathbf{h}(0)^T \mathbf{h}(1)^T \cdots \mathbf{h}(N \Leftrightarrow 1)^T]^T$ and

$$\mathbf{D}_{DFT}^{\{k,p\}} = \text{blockdiag}[\mathbf{I}_p | e^{-\jmath\frac{2\pi k}{m}}\mathbf{I}_p | \cdots | e^{-\jmath\frac{2\pi(N-1)k}{m}}\mathbf{I}_p] \tag{6}$$

4 Gladyshev's Theorem And Miamee Process

Gladyshev's theorem [6] states that :

Theorem 1 *Function $\mathbf{R}_{xx}(n,\tau)$ is the correlation function of some PCS (Periodically Correlated Sequence) if and only if the matrix-valued function :*

$$\underline{\mathbf{R}}(\tau) = \left[\mathbf{R}_{xx}^{\{kk'\}}(\tau)\right]_{k,k'=0}^{m-1} \tag{7}$$

$$\textit{where } \mathbf{R}_{xx}^{\{kk'\}}(\tau) = \mathbf{R}_{xx}^{\{k-k'\}}(\tau)e^{2\pi\jmath k\tau/m} \tag{8}$$

is the matricial correlation function of some m-variate stationary sequence.

Reminding that $\mathbf{R}_{xx}^{\{k\}}(\tau) = \mathbf{R}_{xx}^{\{m-k\}H}(\Leftrightarrow\tau)$, the following matrix

$$\underline{\mathbf{R}} \triangleq \begin{bmatrix} \underline{\mathbf{R}}(0) & \underline{\mathbf{R}}(1) & \cdots & \underline{\mathbf{R}}(K \Leftrightarrow 1) \\ \underline{\mathbf{R}}(\Leftrightarrow 1) & \underline{\mathbf{R}}(0) & \cdots & \underline{\mathbf{R}}(K \Leftrightarrow 2) \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\mathbf{R}}(1 \Leftrightarrow K) & \underline{\mathbf{R}}(2 \Leftrightarrow K) & \cdots & \underline{\mathbf{R}}(0) \end{bmatrix} \tag{9}$$

is an hermitian $K \times K$ block Toeplitz matrix of $Mm \times Mm$ blocks.

Then, Miamee [9] gives us the explicit expression of the multivariate stationary process associated :

$$\underline{\mathbf{Z}}_n = \left[\mathbf{Z}_n^k\right]_{k=0}^{m-1} \quad \text{where } \mathbf{Z}_n^k = \bigoplus_{j=0}^{m-1} \mathbf{x}(n+j)e^{2\pi\jmath k(n+j)/m} \tag{10}$$

where $\oplus$ is the direct sum, i.e., noting $w = e^{2\pi\jmath/m}$

$$\mathbf{Z}_n^k = w^{kn}[\mathbf{x}(n), \mathbf{x}(n+1)w^k, \cdots, \mathbf{x}(n+m \Leftrightarrow 1)w^{k(m-1)}] \tag{11}$$

is defined in a Hilbert space, where the correlation is the following Euclidean product :

$$< \mathbf{Z}_n^k, \mathbf{Z}_{n+l}^{k'} >= \sum_{j=0}^{m-1} \mathrm{E}\left\{\mathbf{Z}_n^k(j)\mathbf{Z}_{n+l}^{k'}{}^H(j)\right\} \tag{12}$$

and $\underline{\mathbf{Z}}_n = [\mathbf{Z}_n^{0\,T}\mathbf{Z}_n^{1\,T} \cdots \mathbf{Z}_n^{m-1\,T}]^T$ with the classical correlation for multivariate stationary processes.

On the other hand, Miamee gives the link between the linear prediction on $\underline{\mathbf{Z}}_n$ and the cyclic AR model of $\mathbf{x}(n)$, which we will use to derive an efficient way of computing the linear predictor.

5 Expression of $\underline{\mathbf{Z}}_n$ w.r.t. $\mathbf{u}(n)$ and $\mathbf{h}(n)$

From $\mathbf{Z}_n^k = \overset{m-1}{\underset{j=0}{\oplus}} \mathbf{x}(n+j)e^{2\pi \jmath k(n+j)/m}$ and

$$\begin{aligned}
\mathbf{x}(n+j) &= \sum_{k=0}^{L-1} \mathbf{h}(k)\mathbf{u}(n+j \Leftrightarrow k) + \mathbf{v}(n+j) \\
&= \mathbf{H}_N \begin{bmatrix} \mathbf{u}(n+j) \\ \mathbf{u}(n+j+1) \\ \vdots \\ \mathbf{u}(n+j \Leftrightarrow N+1) \end{bmatrix} + \mathbf{v}(n+j)
\end{aligned} \tag{13}$$

Defining $\mathbf{U}_{n+j} = [\mathbf{u}(n+j)^T \cdots \mathbf{u}(n+j \Leftrightarrow N+1)^T]^T$ and $\mathbf{H}_N^{\{k\}} = [w^{-kj}\mathbf{h}(j)]_{j=0}^{N-1}$ we express the Miamee process as :

$$\begin{aligned}
\mathbf{Z}_n^k &= \overset{m-1}{\underset{j=0}{\oplus}} (\mathbf{H}_N^{\{-k\}} w^{kn} \mathbf{U}_{n+j} + \mathbf{v}(n+j)e^{2\pi \jmath k \frac{n+j}{m}}) \\
&= \mathbf{H}_N^{\{-k\}} w^{kn} [\mathbf{U}_n \mathbf{U}_{n+1} \cdots \mathbf{U}_{n+m-1}] + \overset{m-1}{\underset{j=0}{\oplus}} \mathbf{v}(n+j)e^{2\pi \jmath k \frac{n+j}{m}}
\end{aligned} \tag{14}$$

$$\Rightarrow \underline{\mathbf{Z}}_n = \mathbf{H}_{tot}\mathcal{U}(n) + \mathcal{V}(n) \tag{15}$$

where we noted $\mathbf{H}_{tot} = [\mathbf{H}_N^{\{0\}T} \mathbf{H}_N^{\{-1\}T} \cdots \mathbf{H}_N^{\{1-m\}T}]^T$,

$$\mathcal{U}(n) = \mathbf{D}_{DFT}^{\{n,pN\}} [\mathbf{U}_n \mathbf{U}_{n+1} \cdots \mathbf{U}_{n+m-1}] \tag{16}$$

$$\mathcal{V}(n) = \begin{bmatrix} \mathbf{v}(n) & \cdots & \mathbf{v}(n+m \Leftrightarrow 1) \\ \mathbf{v}(n)w^n & \cdots & \mathbf{v}(n+m \Leftrightarrow 1)w^{n+m-1} \\ \vdots & \ddots & \vdots \\ \mathbf{v}(n)w^{n(m-1)} & \cdots & \mathbf{v}(n+m \Leftrightarrow 1)w^{(m-1)(n+m-1)} \end{bmatrix} \tag{17}$$

$$\Rightarrow \underline{\mathbf{Z}} = \mathcal{T}_{L+N-1}(\mathbf{H}_{tot})\mathcal{U}_L + \mathcal{V}_L \tag{18}$$

where $\mathcal{U}_L = [\mathcal{U}(n)]_{n=L-1}^0$ clearly is a stationary process whose correlation matrix can easily be deduced from $\mathbf{R}_{aa}$.

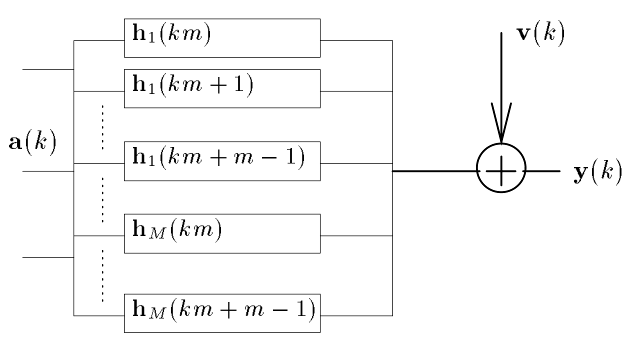

Figure 3: (Classical) Time Series Channel model representation

Based on relation (18), we apply the classical subspace fitting and linear prediction channel identification schemes, as detailed below.

6 Identifiability

Provided the data collected are numerous enough, the rank condition on $\mathcal{T}_{L+N-1}(\mathbf{H}_{tot})$ lead to the usual identifiability conditions, i.e. that $\mathbf{H}_{tot}(z)$ must be minimum phase which is equivalent to the condition that $\mathbf{h}_i(z)$ may not have $\frac{2\pi}{m}$ equispaced zeros and that $\mathbf{h}_i(z)$ and $\mathbf{h}_j(z)$ may not have common zeros for all $i \neq j$.

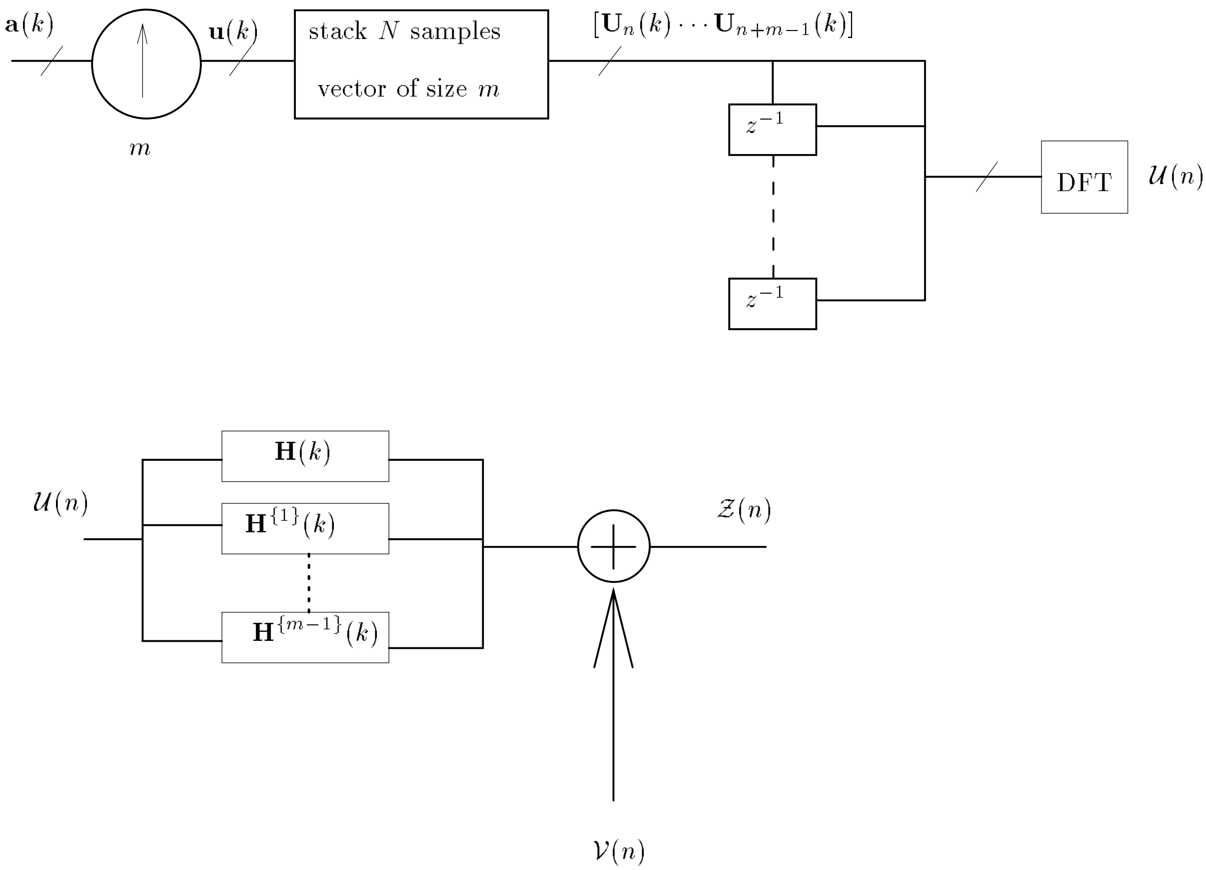

Figure 4: New Channel model

7 Signal Subspace Fitting

We recall briefly the signal subspace fitting [11, 1] (noise subspace based) blind channel identification algorithm here under.

One can write the (compact form of the) SVD of the cyclocorrelation matrix $\underline{\mathbf{R}} = \mathbf{ABC}^H$ with the relations:

$$\text{range}\{\mathbf{A}\} = \text{range}\{\mathbf{C}\} = \text{range}\{\mathcal{T}_K(\mathbf{H}_{tot})\} \tag{19}$$

We can then solve the classical subspace fitting problem :

$$\min_{\mathbf{H}_{tot},T} ||\mathcal{T}_K(\mathbf{H}_{tot}) \Leftrightarrow \mathbf{A}T||_F^2 \tag{20}$$

If we introduce $\mathbf{A}^\perp$ such that $[\mathbf{A}\mathbf{A}^\perp]$ is a unitary matrix, this leads to

$$\min_{\mathbf{H}_{tot}} \mathbf{H}_{tot}^t \left[\sum_{i=D^\perp}^{KMm} \mathcal{T}_N(\mathbf{A}_i^{\perp Ht}) \mathcal{T}_N^H(\mathbf{A}_i^{\perp Ht}) \right] \mathbf{H}_{tot}^{Ht} \tag{21}$$

where $\mathbf{U}_i^\perp$ is a $KMm^2 \times 1$, $D^\perp = N + K$ and superscript ^{t} denotes the transposition of the blocks of a block matrix. Under constraint $||\mathbf{H}_{tot}|| = 1$, $\hat{\mathbf{H}}_{tot}^t$ is then the eigenvector corresponding to the minimum eigenvalue of the matrix between brackets. Similar work was done by Schell in [4] for the Direction of Arrival estimation problem. One can lower the computational burden by using $D^\perp > N + K$ (see a.o. [11]), which leads to loss of performance. A reduced complexity signal subspace fitting without loss of performance is described in [3].

The case $p > 1$ can be (partially) solved in a manner similar to [12] and [8].

8 Linear Prediction

We consider the denoised case. The correlation matrix is then computed as follows : $\mathbf{R}^{\{0\}}_{xx,sb} = \mathbf{R}^{\{0\}}_{xx} \Leftrightarrow \mathbf{R}_{\mathcal{V}\mathcal{V}}(\tau)$ yields :

$$
\begin{aligned}
[\mathbf{R}_{\mathcal{V}\mathcal{V}}(\tau)]_{i,j} \quad &= \sum_{l=0}^{m-1} \mathrm{E}\left\{\mathbf{v}(n+l)\mathbf{v}^H(n+l+\tau)\right\} w^{i(n+l)} w^{-j(n+l+\tau)} \\
&= \mathbf{R}_{vv}(\tau) w^{n(i-j)-j\tau} \sum_{l=0}^{m-1} w^{(i-j)l} \\
&= \mathbf{R}_{vv}(\tau) w^{n(i-j)-j\tau} m\delta_{ij} = m\delta_{ij}\mathbf{R}_{vv}(\tau) w^{-j\tau}
\end{aligned}
\tag{22}
$$

Hence $\mathbf{R}_{\mathcal{V}\mathcal{V}}(\tau) = \mathbf{R}_{vv}(\tau)$ blockdiag$[\mathbf{I}_M|w^{\tau}\mathbf{I}_M|w^{2\tau}\mathbf{I}_M|\cdots|w^{(m-1)\tau}\mathbf{I}_M]$, which, in $\underline{\mathbf{R}}$, corresponds to the noise contribution of the zero cyclic frequency cyclic correlation.

From equation (18) and noting $\mathcal{Z}_K(n \Leftrightarrow 1) = [\underline{\mathbf{Z}}_j]_{j=n-1}^{j=n-K}$, the predicted quantities are :

$$\hat{\underline{\mathbf{Z}}}(n)|_{\mathcal{Z}_K(n-1)} = \mathbf{p}_1\underline{\mathbf{Z}}_{n-1} + \cdots + \mathbf{p}_K\underline{\mathbf{Z}}_{n-K} \tag{23}$$

$$\tilde{\underline{\mathbf{Z}}}(n) = \underline{\mathbf{Z}}(n) \Leftrightarrow \hat{\underline{\mathbf{Z}}}(n)|_{\mathcal{Z}_K(n-1)} \tag{24}$$

Following [16], we rewrite the correlation matrix as

$$\underline{\mathbf{R}} = \begin{bmatrix} \mathbf{R}_o & \mathbf{r}_K \\ \mathbf{r}_K^H & \mathbf{R}_{K-1} \end{bmatrix} \tag{25}$$

this yields the prediction filter :

$$\mathbf{P}_K \stackrel{\triangle}{=} [\mathbf{p}_1 \cdots \mathbf{p}_K] = \Leftrightarrow \mathbf{r}_K \mathbf{R}_{K-1}^{-1} \tag{26}$$

and the prediction error variance :

$$\sigma_{\tilde{\underline{\mathbf{Z}}},K} = \mathbf{R}_o \Leftrightarrow \mathbf{P}_K\mathbf{r}_K^H = \mathbf{H}_{tot}(0)\sigma^2_{\mathcal{U},K+N-1}\mathbf{H}^H_{tot}(0) \tag{27}$$

where the inverse might be replaced by the Moore-Penrose pseudo-inverse, and still yield a consistent channel estimate. Another way of being robust to order overestimation would be to use the Levinson-Wiggins-Robinson (LWR) algorithm to find the prediction quantities and estimate the order with this algorithm.

Lots of ways are possible to go from the prediction quantities to the channel estimate ([13] and [2]). We used the optimal solution here under.

For $K = \underline{K} = \left\lceil \dfrac{N \Leftrightarrow 1}{M.m \Leftrightarrow 1} \right\rceil$, (27) allows us to find $\mathbf{H}_{tot}(0)$ up to a scalar multiple. Let $\mathbf{H}^{\perp}_{tot}$ be $M.m \times (M.m \Leftrightarrow 1)$ of rank $M.m \Leftrightarrow 1$ such that $\mathbf{H}^{\perp}_{tot}\mathbf{H}_{tot}(0) = 0$, then

$$\mathbf{F}^b_{\underline{K}+1} = \mathbf{H}^{\perp H}_{tot}\mathbf{P}_{\underline{K}} \tag{28}$$

is a set of $M.m \Leftrightarrow 1$ blocking equalizers, since $\mathbf{F}^b\underline{\mathbf{Z}} = 0$. Due to the commutativity of convolution, we find :

$$\mathbf{F}^b_K \mathcal{T}_K(\mathbf{H}_{tot}) = 0 \Leftrightarrow \mathbf{H}^t_{tot}\mathcal{T}_N(\mathbf{F}^{bt}_L) \tag{29}$$

Now

$$\dim\left(\mathrm{Range}^{\perp}\left\{\mathcal{T}_N(\mathbf{F}^{bt}_{\underline{K}+1})\right\}\right) = 1 \tag{30}$$

so that we can identify the channel $\mathbf{H}^{tH}_{tot}$ as the last right singular vector of $\mathcal{T}_N(\mathbf{F}^{bt}_{\underline{K}+1})$.

9 Computational Aspects

It is obvious that the correlation matrix $\underline{\mathbf{R}}$ built from the cyclic correlations is bigger (in fact each scalar in $\mathbf{R}$ is replaced by a $m \times m$ block in $\underline{\mathbf{R}}$) than the corresponding matrix built from the classical Time Series representation of oversampled stationary signals. This fact must be balanced with the stronger structure that is cast in our correlation matrix. In fact, one can show that the estimates $\hat{\mathbf{H}}_N^{\{-k\}}$ are strictly related (i.e. $\hat{\mathbf{H}}_N^{\{-k\}} = [w^{-kj}\hat{\mathbf{h}}(j)]_{j=0}^{N-1}$ for all k), which indicates us that this structure should lead to reduced complexity algorithms w.r.t. the original ones. When developing the expressions in detail, this is particularly obvious in linear prediction, where the prediction filter has some strong structure (which is also visible in [10]). Moreover, as noted in [15], the multichannel linear prediction problems correspond to a block triangular factorization and to an orthogonalization of the block components of the vector $\underline{\mathbf{Z}}$.

Coming back to the original channel model, we can alternatively introduce sequential process in the orthogonalization process and orthogonalize the elements of the vector $\mathbf{X} = [\mathbf{x}(n) \cdots \mathbf{x}(n+K)]$ scalar component by scalar component. the elements of the vector $\mathbf{X} = [\mathbf{x}(n) \cdots \mathbf{x}(n+K)]$ This leads to the cyclic prediction filters, whose explicit relations to the multivariate predictions filters are known, and results from a true (non-block) triangular factorization.

10 Simulations

In our simulations, we restrict ourselves to the $p = 1$ case, using a randomly generated real channel of length 6T, an oversampling factor of $m = 3$ and $M = 3$ antennas. We draw the NRMSE of the channel, defined as

$$\mathrm{NRMSE} = \sqrt{\frac{1}{100}\sum_{l=1}^{100} ||\hat{\mathbf{h}}^{(l)} \Leftrightarrow \mathbf{h}||_F^2 / ||\mathbf{h}||_F^2} \tag{31}$$

where $\hat{\mathbf{h}}^{(l)}$ is the estimated channel in the l^{th} trial.

The correlation matrix is calculated from a burst of 100 QAM-4 symbols (note that if we used real sources, we would have used the conjugate cyclocorrelation, which is another means of getting rid of the noise, provided it is circular). For these simulations, we used 100 Monte-Carlo runs.

10.1 Subspace fitting

The estimations of 25 realizations, for an SNR of 20 dB, are reproduced here under.

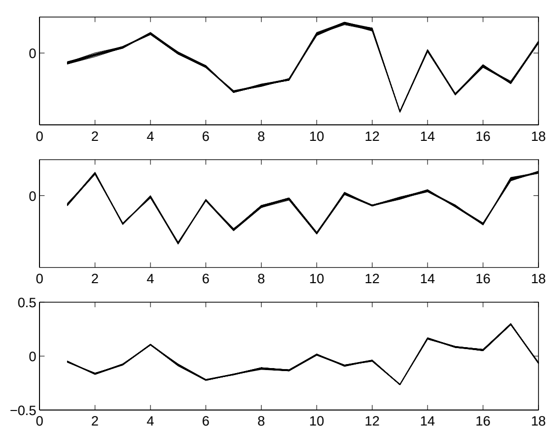

Figure 5: Simple channel estimate

For comparison, we used the same algorithm for the classical Time Series representation of the oversampled signal. The results here under show a better performance for the classic approach, which is due to the fact that we used the same complexity for both algorithms (same matrix size), which results in a lower noise subspace size for the cyclic approach. In theory, when one uses the same subspace size, as there is a one to one correspondence between the elements of the classic correlation matrix and the elements of the cyclic correlation matrix, the performances should be equal. The third curve illustrates this fact.

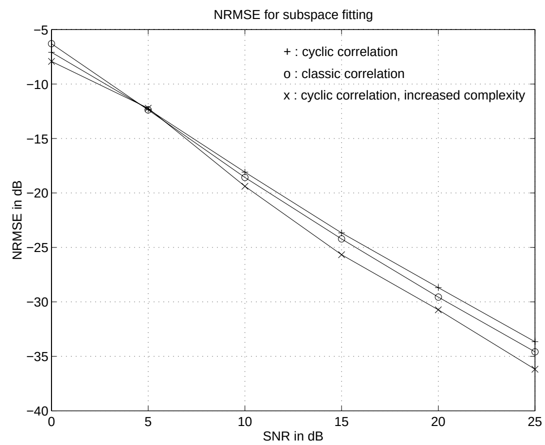

Figure 6: Subspace fitting estimation error

In more realistic cases, when one uses a channel model where the transmitter and receiver filters are incorporated, the cyclic approach gives better results. This is mostly due to the fact that we can better refine the channel length estimate (and should be preceded by a good channel length estimation algorithm). Indeed, if we use a channel of the form (where $M = 2$ and $m = 2$)

$$\begin{pmatrix} \epsilon & * & * & * & * & * & \epsilon \\ \epsilon & * & * & * & * & * & \epsilon \end{pmatrix} \tag{32}$$

where ϵ is a near zero value, the cyclocorrelation approach can afford to restrict to the central part of the channel, but the classical approach will try to find the $M * m$ multichannel :

$$\begin{pmatrix} \epsilon & * & * \\ * & * & * \\ \epsilon & * & * \\ * & * & * \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} * & * & * \\ * & * & \epsilon \\ * & * & * \\ * & * & \epsilon \end{pmatrix} \tag{33}$$

with 2 more (near zero) parameters to estimate, which will globally give a worse estimation. Following figures illustrate, for moderate SNR, the performance enhancement for a 5T channel combined to a 90% excess bandwidth raised cosine filter (we continue to use $M = 3$ and $m = 3$).

10.2 Linear prediction

For the linear prediction, we expect to have a slightly better performance in the cyclic approach than in the classic approach. Indeed, in the classic approach, if we use for example $M = 1$ antenna and an oversampling factor of $m = 3$, we predict $[x(n)x(n \Leftrightarrow 1)x(n \Leftrightarrow 2)]^T$ based on $[x(n \Leftrightarrow 3)x(n \Leftrightarrow 4)\cdots]^T$, whereas in the cyclic approach we predict the scalar $x(n)$ based on $[x(n \Leftrightarrow 1)x(n \Leftrightarrow 2)x(n \Leftrightarrow 3)\cdots]^T$. The corresponding prediction filter thus captures little more prediction features in the cyclic case.

On the other hand, the noise contribution being only present in the zero cyclic frequency cyclic correlation (see equation 5), we expect a better behavior of the method if we do not take the noise into account in the correlation matrix (i.e. we don't estimate the noise variance before

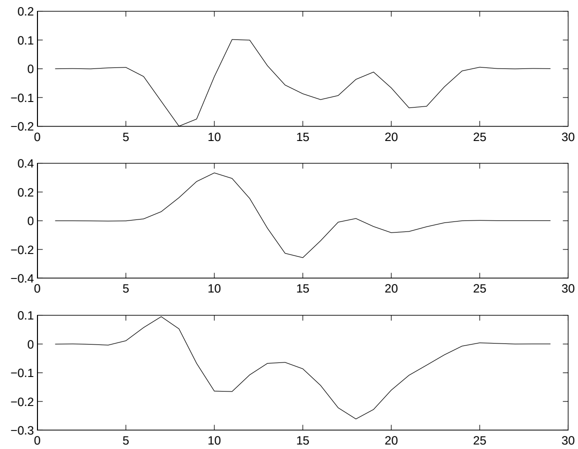

Figure 7: Combined channel and transmission/reception filter

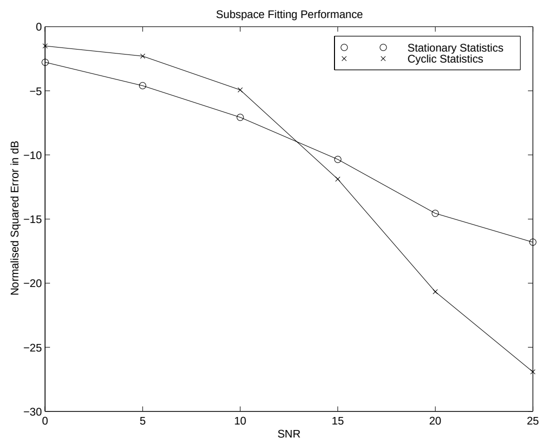

Figure 8: Subspace fitting estimation error

doing the linear prediction). Those expectations are confirmed by the following simulations, note that the mention LP on cyclic statistics refers to the use of $\underline{\mathbf{R}}$ where the noise contribution has been removed, whereas the mention LP on cyclic statistics, no "denoising" refers to the use of the plain correlation matrix.

11 Conclusions

Using the stationary multivariate representation introduced by [6] and [9] [10], we have explicitly expressed this process. It can be seen as the output of a system with transfer channel $\mathbf{H}_{tot} = [\mathbf{H}_N^{\{0\}T}\mathbf{H}_N^{\{-1\}T}\cdots\mathbf{H}_N^{\{1-m\}T}]^T$ and input easily related to the actual system input. Once these quantities expressed, application of the classical subspace fitting and linear prediction algorithms is straightforward.

For the subspace fitting, one has essentially the same performance as in the Time Series Representation [16]. The only advantage one could expect is some refinement in the channel order estimation prior to the subspace fitting. The main drawback is the increase of the computational burden.

For the linear prediction, we get a better performance due to the fact that we take the very

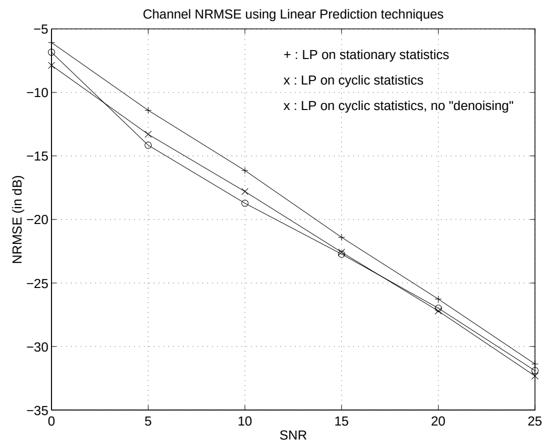

Figure 9: Linear prediction estimation error

near past into account. What is more, use of modular multichannel linear prediction algorithms like those described in [7] provide fast algorithms and adaptive implementations.

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