Symmetric two-user MIMO BC with Evolving Feedback

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***Abstract*—Extending recent findings on the two-user MISO broadcast channel (BC) with imperfect and delayed channel state information at the transmitter (CSIT), the work here explores the performance of the two user MIMO BC, in the presence of feedback with evolving quality and timeliness. Under standard assumptions, and in the presence of M antennas at the transmitter and N antennas per receiver, the work derives the DoF region, which is optimal for a large range of current and delayed CSIT quality. This region concisely captures the effect of having predicted, current and delayed-CSIT, as well as concisely captures the effect of the quality of CSIT offered at any time, about any channel realization. In addition to the progress towards describing the limits of using such imperfect and delayed feedback in MIMO settings, the work offers different insights that include the fact that DoF optimality, in the presence of an increased number of receiving antennas, can be achieved with reduced quality feedback.**

I. Introduction

A. MIMO BC model

For the setting of the multiple-input multiple-output broadcast channel (MIMO BC), we consider the case where an M antenna transmitter, sends information to two receivers with N receive antennas each. In this setting, the received signals at the two receivers take the form

$$\boldsymbol{y}_t^{(1)} = \boldsymbol{H}_t^{(1)} \boldsymbol{x}_t + \boldsymbol{z}_t^{(1)} \quad (1a)$$

$$\boldsymbol{y}_t^{(2)} = \boldsymbol{H}_t^{(2)} \boldsymbol{x}_t + \boldsymbol{z}_t^{(2)} \quad (1b)$$

where $\boldsymbol{H}_t^{(1)} \in \mathbb{C}^{N\times M}, \boldsymbol{H}_t^{(2)} \in \mathbb{C}^{N\times M}$ respectively represent the first and second receiver channels at time t, where $\boldsymbol{z}_t^{(1)}, \boldsymbol{z}_t^{(2)}$ represent unit power AWGN noise at the two receivers, and where $\boldsymbol{x}_t \in \mathbb{C}^{M\times 1}$ is the input signal with power constraint $\mathbb{E}[||\boldsymbol{x}_t||^2] \leq P$.

B. Degrees-of-freedom as a function of feedback quality

In the presence of perfect channel state information at the transmitter (CSIT), the degrees-of-freedom (DoF) performance[1] for the case of the MIMO BC, is given by (cf. [1])

$$\{d_1 \leq \min\{M,N\}, \ d_2 \leq \min\{M,N\}, \quad (2)$$

$$d_1 + d_2 \leq \min\{M, 2N\}\}. \quad (3)$$

In the absence of any CSIT though, this performance reduces, from that in (2),(3), to the DoF region

$$\{d_1 + d_2 \leq \min\{M,N\}\} \quad (4)$$

corresponding to a symmetric DoF corner point $(d_1 = d_2 = \min\{M,N\}/2)$ (cf. [2], [3]).

This gap necessitates the use of imperfect and delayed feedback, as this was studied in works like [4]–[18] for specific instances. The work here makes progress towards describing the limits of this use of imperfect and delayed feedback.

C. Predicted, current and delayed CSIT

As in [19], we consider communication of an infinite duration n.

For the case of the BC, we consider a random fading process $\{\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}\}_{t=1}^n$, and a feedback process that provides CSIT estimates $\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t,t'=1}^n$ (of channel $\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}$) at any time $t' = [1, \cdots, n]$. For the channel $\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}$ at a specific time t, the set of all available estimates $\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t'}$, can be naturally split in the *predicted estimates* $\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t'<t}$ that are offered before the channel materializes, the *current estimate* $\hat{\boldsymbol{H}}_{t,t}^{(1)}, \hat{\boldsymbol{H}}_{t,t}^{(2)}$ at time t, and the *delayed estimates* $\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t'>t}$ that may allow for retrospective compensation for the lack of perfect quality feedback. Naturally the fundamental measure of feedback quality is given by the precision of estimates at any time about any channel, i.e., is given by

$$\{(\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t'}^{(1)}), (\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t'}^{(2)})\}_{t,t'=1}^n. \quad (5)$$

These estimation-error sets of course fluctuate depending on the instance of the problem, and as expected, the overall optimal performance is defined by the statistics of the above estimation errors. We here only assume that these errors have zero-mean circularly-symmetric complex Gaussian entries, that are *spatially* uncorrelated, and that at any time t, the current estimation *error* is independent of the channel estimates up to that time.

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[1]We remind the reader that in the high-SNR setting of interest, for an achievable rate pair (R_1, R_2) for the first and second receiver respectively, the corresponding DoF pair (d_1, d_2) is given by $d_i = \lim_{P\to\infty} \frac{R_i}{\log P}, \ i = 1, 2$ and the corresponding DoF region is then the set of all achievable DoF pairs.

D. Notation, conventions and assumptions

We will generally follow the notations and assumptions in [19], and will adapt them to the MIMO setting. We will use the notation

$$\alpha_t^{(i)} \triangleq -\lim_{P\to\infty} \frac{\mathbb{E}[||\boldsymbol{H}_t^{(i)} - \hat{\boldsymbol{H}}_{t,t}^{(i)}||_F^2]}{\log P}, \quad (6)$$

$$\beta_t^{(i)} \triangleq -\lim_{P\to\infty} \frac{\mathbb{E}[||\boldsymbol{H}_t^{(i)} - \hat{\boldsymbol{H}}_{t,t+\eta}^{(i)}||_F^2]}{\log P} \quad (7)$$

where $\alpha_t^{(i)}$ is used to describe the *current quality exponent* for the CSIT for channel $\boldsymbol{H}_t^{(i)}$ of receiver $i,\ i=1,2$, while $\beta_t^{(i)}$ is used to describe the *delayed quality exponents* for each user. In the above, η can be as large as necessary, but it must be finite, as we here consider delayed CSIT that arrives after a finite delay from the channel it describes. The above used $||\bullet||_F$ to denote the Frobenius norm of a matrix.

As argued in [19], the results in [20], [21] easily show that without loss of generality, in the DoF setting of interest, we can restrict our attention to the range

$$0 \le \alpha_t^{(i)} \le \beta_t^{(i)} \le 1. \quad (8)$$

Here having $\alpha_t^{(1)} = \alpha_t^{(2)} = 1$, corresponds to the highest quality CSIT with perfect timing (full CSIT) for the specific channel at time t, while having $\beta_t^{(i)} = 1$ corresponds to having perfect delayed CSIT for the same channel, i.e., it corresponds to the case where at some point $t' > t$, the transmitter has perfect estimates of the channel that materialized at time t.

Furthermore we will use the notation

$$\bar{\alpha}^{(i)} \triangleq \lim_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} \alpha_t^{(i)}, \quad \bar{\beta}^{(i)} \triangleq \lim_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} \beta_t^{(i)}, \quad i=1,2 \quad (9)$$

to denote the average of the quality exponents. As in [19] we will adopt the mild assumption that any sufficiently long subsequence $\{\alpha_t^{(1)}\}_{t=\tau}^{\tau+T}$ (resp. $\{\alpha_t^{(2)}\}_{t=\tau}^{\tau+T}, \{\beta_t^{(1)}\}_{t=\tau}^{\tau+T}, \{\beta_t^{(2)}\}_{t=\tau}^{\tau+T}$) has an average that converges to the long term average $\bar{\alpha}^{(1)}$ (resp. $\bar{\alpha}^{(2)}, \bar{\beta}^{(1)}, \bar{\beta}^{(2)}$), for any τ and for some finite T that can be chosen to be sufficiently large to allow for the above convergence.

Implicit in our definition of the quality exponents, is our assumption that $\mathbb{E}[||\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t'}^{(1)}||_F^2] \le \mathbb{E}[||\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t''}^{(1)}||_F^2],\ \mathbb{E}[||\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t'}^{(2)}||_F^2] \le \mathbb{E}[||\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t''}^{(2)}||_F^2]$, for any $t' > t''$, which simply reflects the fact that - if necessary - one can revert back to past estimates of possibly better quality statistics. This assumption can be removed - after a small change in the definition of the quality exponents - without an effect to the main result.

Throughout this paper, $(\bullet)^{\mathsf{T}}$ and $(\bullet)^{\mathsf{H}}$ will denote the transpose and conjugate transpose of a matrix respectively, while $\mathrm{diag}(\bullet)$ will denote a diagonal matrix, $||\bullet||$ will denote the Euclidean norm, and $|\bullet|$ will denote the magnitude of a scalar. $o(\bullet)$ comes from the standard Landau notation, where $f(x) = o(g(x))$ implies $\lim_{x\to\infty} f(x)/g(x) = 0$. We also use $\doteq$ to denote *exponential equality*, i.e., we write $f(P) \doteq P^B$ to denote $\lim_{P\to\infty} \frac{\log f(P)}{\log P} = B$. Similarly $\dot{\ge}$ and $\dot{\le}$ will denote exponential inequalities. Logarithms are of base 2. $(\bullet)^+ = \max\{\bullet, 0\}$.

Furthermore we adhere to the common convention (see [9], [22]–[25]) of assuming perfect and global knowledge of channel state information at the receivers (perfect global CSIR), where the receivers know all channel states and all estimates. We will also adopt the common convention (see [24]–[27]) of assuming that the current estimation error is statistically independent of current and past estimates. A discussion on this can be found in [19] which argues that this assumption fits well with many channel models, spanning from the fast fading channel (i.i.d. in time), to the correlated channel model as this is considered in [26], to the quasi-static block fading model where the CSIT estimates are successively refined while the channel remains static. Additionally we consider the entries of each estimation error matrix $\boldsymbol{H}_t^{(i)} - \hat{\boldsymbol{H}}_{t,t'}^{(i)}$ to be i.i.d. Gaussian [2]. Finally we will refer to a CSIT process with *'sufficiently good delayed CSIT'*, to be a process for which $\min\{\bar{\beta}^{(1)}, \bar{\beta}^{(2)}\} \ge \min\{1, M - \min\{M,N\}, \frac{N(1+\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)})}{\min\{M,2N\}+N}, \frac{N(1+\bar{\alpha}^{(2)})}{\min\{M,2N\}}\}$.

E. Existing results directly relating to the current work

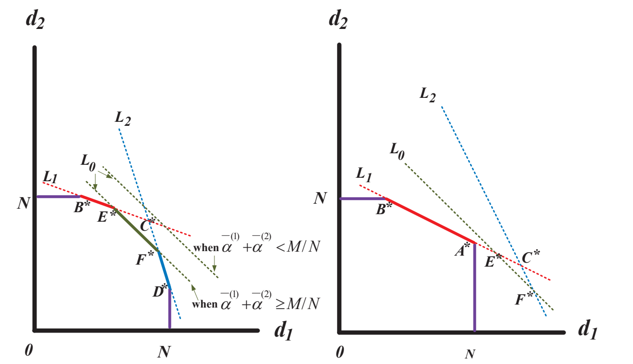
The work here builds on the ideas of [22] on using delayed CSIT to retrospectively compensate for interference due to lack of current CSIT, on the ideas in [26] and later in [24], [25] on exploiting perfect delayed and imperfect current CSIT, as well as the work in [28], [29] which - in the context of imperfect and delayed CSIT - introduced encoding and decoding with a phase-Markov structure that will be used later on. The work here is also motivated by the work in [8] which considered the use of delayed feedback in different MIMO BC settings, as well as by recent progress in [9] that considered MIMO BC and MIMO IC settings that enjoyed perfect delayed feedback as well as imperfect current feedback of a quality that remained unchanged throughout the communication process ($\alpha^{(1)} = -\lim_{P\to\infty} \frac{\mathbb{E}[||\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t}^{(1)}||_F^2]}{\log P}, \alpha^{(2)} = -\lim_{P\to\infty} \frac{\mathbb{E}[||\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t}^{(2)}||_F^2]}{\log P},\ \forall t$). The work is finally motivated by the recent approach in [19] that employed sequences of evolving quality exponents to address the fundamental problem of deriving the performance limits given a general CSIT process of a certain quality.

II. DoF REGION OF THE MIMO BC

In the following we present the DoF region for any CSIT process with sufficiently good delayed CSIT. After the main theorem, and different corollaries, we will present a brief sketch of the encoding part of the transceiver, that allows for the inner bound. The corresponding outer bound is found in the journal version [31] of this current work (see also our work in [30] for the outer bound in the presence of statistical symmetric of CSIT quality across the two users).

We recall that we consider communication of large duration n, a possibly correlated channel process $\{\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}\}_{t=1}^{n}$,

[2] We here make it clear that we are simply referring to the MN entries in each such specific matrix $\boldsymbol{H}_t^{(i)} - \hat{\boldsymbol{H}}_{t,t'}^{(i)}$, and that we certainly do not suggest that the error entries are i.i.d. in time or across users.



(a) Case: $N(1+\bar{\alpha}^{(2)})/M \geq \bar{\alpha}^{(1)}$ and $M>N$ **(b) Case:** $N(1+\bar{\alpha}^{(2)})/M \leq \bar{\alpha}^{(1)}$ and $M>N$

Fig. 1. Optimal DoF regions, for two different cases, with $M > N$ and $\min\{\bar{\beta}^{(1)}, \bar{\beta}^{(2)}\} \geq \min\{1, M - \min\{M,N\}, \frac{N(1+\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)})}{\min\{M,2N\}+N}, \frac{N(1+\bar{\alpha}^{(2)})}{\min\{M,2N\}}\}$. The corner points take the following values: $A^* = \big(N, \frac{(M-N)N(1+\bar{\alpha}^{(2)})}{M}\big)$, $B^* = \big((M-N)\bar{\alpha}^{(2)}, N\big)$, $C^* = \big(\frac{MN}{M+N}(1+\bar{\alpha}^{(1)} - \frac{N}{M}\bar{\alpha}^{(2)}), \frac{MN}{M+N}(1+\bar{\alpha}^{(2)} - \frac{N}{M}\bar{\alpha}^{(1)})\big)$, $D^* = \big(N, (M-N)\bar{\alpha}^{(1)}\big)$, $E^* = \big(M-N\bar{\alpha}^{(2)},\ N\bar{\alpha}^{(2)}\big)$, $F^* = \big(N\bar{\alpha}^{(1)},\ M - N\bar{\alpha}^{(1)}\big)$. Line L_0 corresponds to the bound in (12), Line L_1 corresponds to the bound in (14), while line L_2 corresponds to the bound in (13).

and a feedback process of quality defined by the statistics of $\{(\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t'}^{(1)}), (\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t'}^{(2)})\}_{t=1,t'=1}^n$. We henceforth, without loss of generality, label the users so that $\bar{\alpha}^{(2)} \leq \bar{\alpha}^{(1)}$.

Theorem 1: The optimal DoF region of the two-user $(M \times (N,N))$ MIMO BC with a CSIT process $\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t=1,t'=1}^n$ of quality $\{(\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t'}^{(1)}), (\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t'}^{(2)})\}_{t=1,t'=1}^n$ that has sufficiently good delayed CSIT, is given by

$$d_1 \leq \min\{M,N\} \tag{10}$$

$$d_2 \leq \min\{M,N\} \tag{11}$$

$$d_1 + d_2 \leq \min\{M,2N\} \tag{12}$$

$$\frac{d_1}{\min\{M,N\}} + \frac{d_2}{\min\{M,2N\}} \leq 1 + \frac{\min\{M,2N\} - \min\{M,N\}}{\min\{M,2N\}}\ \bar{\alpha}^{(1)} \tag{13}$$

$$\frac{d_1}{\min\{M,2N\}} + \frac{d_2}{\min\{M,N\}} \leq 1 + \frac{\min\{M,2N\} - \min\{M,N\}}{\min\{M,2N\}}\ \bar{\alpha}^{(2)}. \tag{14}$$

A. Imperfect current CSIT can be as useful as perfect current CSIT

The above results allow for direct conclusions on the amount of CSIT that is necessary to achieve the optimal DoF performance associated to perfect and immediately available CSIT. We recall that there is no need for CSIT when $M \leq N$. The proofs for the following corollary, and of the corollary immediately after that, are direct from the above theorems.

Corollary 1a: A CSIT process that offers

$$\bar{\alpha}^{(1)} + \bar{\alpha}^{(2)} \geq \min\{M,2N\}/N$$

can achieve the same optimal sum-DoF as the process that has perfect and immediately available CSIT ($\bar{\alpha}^{(1)} = \bar{\alpha}^{(2)} = 1$).

The above suggests that increasing the number of receive antennas, can have the added benefit - in addition to an increased optimal DoF - of allowing for a reduction in the required feedback quality $\bar{\alpha}^{(1)}, \bar{\alpha}^{(2)}$, without a DoF penalty.

Along the same lines, the following describes the amount of delayed CSIT that suffices to achieve the DoF associated to perfect delayed CSIT.

Corollary 1b: Any CSIT process that offers $\min\{\bar{\beta}^{(1)}, \bar{\beta}^{(2)}\} \geq \min\{1, M - \min\{M,N\}, \frac{N(1+\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)})}{\min\{M,2N\}+N}, \frac{N(1+\bar{\alpha}^{(2)})}{\min\{M,2N\}}\}$ can achieve the same DoF region as a CSIT process that offers perfect delayed CSIT ($\bar{\beta}^{(1)} = \bar{\beta}^{(2)} = 1$).

III. Phase-Markov transceiver for imperfect and delayed feedback

We proceed to extend the MISO BC scheme in [19], to the current MIMO setting. We here focus on sketching the general structure of the encoding scheme, without presenting all the details. The journal version of this work in [31] provides the details of the process of calibrating the scheme to achieve the different DoF corner points of Theorem 1, as well as the details of decoding.

Before proceeding with the schemes, we again note that we only need to consider the case where $N < M \leq 2N$ simply because the optimal DoF can be achieved without any CSIT whenever $M \leq N$, while having $M > 2N$ can be shown to be equivalent, in terms of DoF, with the case of having $M = 2N$.

The challenge here will be to design a scheme of large duration n, that utilizes the CSIT process $\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t=1,t'=1}^n$. As in [19], the causal scheme will not require knowledge of future quality exponents, nor of predicted CSIT estimates of future channels. We remind the reader that the users are labeled so that $\bar{\alpha}^{(2)} \leq \bar{\alpha}^{(1)}$.

For notational convenience, we will use

$$\hat{\boldsymbol{H}}_t^{(1)} \triangleq \hat{\boldsymbol{H}}_{t,t}^{(1)}, \quad \hat{\boldsymbol{H}}_t^{(2)} \triangleq \hat{\boldsymbol{H}}_{t,t}^{(2)} \tag{15}$$

$$\check{\boldsymbol{H}}_t^{(1)} \triangleq \hat{\boldsymbol{H}}_{t,t+\eta}^{(1)}, \quad \check{\boldsymbol{H}}_t^{(2)} \triangleq \hat{\boldsymbol{H}}_{t,t+\eta}^{(2)} \tag{16}$$

to denote the current and delayed estimates of $\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}$, with the corresponding estimation errors being

$$\tilde{\boldsymbol{H}}_t^{(1)} \triangleq \boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_t^{(1)}, \quad \tilde{\boldsymbol{H}}_t^{(2)} \triangleq \boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_t^{(2)} \tag{17}$$

$$\ddot{\boldsymbol{H}}_t^{(1)} \triangleq \boldsymbol{H}_t^{(1)} - \check{\boldsymbol{H}}_t^{(1)}, \quad \ddot{\boldsymbol{H}}_t^{(2)} \triangleq \boldsymbol{H}_t^{(2)} - \check{\boldsymbol{H}}_t^{(2)}. \tag{18}$$

We will also use the notation

$$P_t^{(\boldsymbol{e})} \triangleq \mathbb{E}|\boldsymbol{e}_t|^2 \tag{19}$$

to denote the power of a symbol $\boldsymbol{e}_t$ corresponding to time-slot t, and we will use $r_t^{(\boldsymbol{e})}$ to denote the prelog factor of the number of bits $r_t^{(\boldsymbol{e})} \log P - o(\log P)$ carried by symbol $\boldsymbol{e}_t$ at time t.

A. Sketch of the encoding process

As in [19], we subdivide the overall time duration n, into S phases, each of duration of T, such that each phase s ($s = 1, 2, \cdots, S$) takes place over the time slots $t \in \mathcal{B}_s$

$$\mathcal{B}_s = \{\mathcal{B}_{s,\ell} \triangleq (s-1)2T + \ell\}_{\ell=1}^{T}, \quad s = 1, \cdots, S. \tag{20}$$

Naturally in the gap of what we define here to be consecutive phases, another message is sent, using the same exact scheme. Going back to the aforementioned assumption, T is sufficiently large so that

$$\frac{1}{T}\sum_{t\in\mathcal{B}_s}\alpha_t^{(i)} \to \bar{\alpha}^{(i)},\ \frac{1}{T}\sum_{t\in\mathcal{B}_s}\beta_t^{(i)} \to \bar{\beta}^{(i)},\ s = 1, \cdots, S \tag{21}$$

$i = 1, 2$. For notational convenience we will also assume that $T > \eta$ (cf. (6)), although this assumption can be readily removed, as this was argued in [19]. Finally with n being infinite, S is also infinite.

Adhering to a phase-Markov structure which - in the context of imperfect and delayed CSIT, was first introduced in [28], [29] - the scheme will quantize the accumulated interference of a certain phase s, broadcast it to both receivers over phase $(s+1)$, while at the same time it will send extra information to both receivers in phase s, which will help recover the interference accumulated in phase $(s-1)$.

We first describe the encoding for all phases except the last phase which will be addressed separately due to its different structure.

1) Phase s, for $s = 1, 2, \cdots, S-1$: In each phase, the scheme combines zero forcing and superposition coding, power and rate allocation, and interference quantizing and broadcasting. We proceed to describe these steps.

a) Zero forcing and superposition coding: At time $t \in \mathcal{B}_s$ (of phase s), the transmitter sends

$$\boldsymbol{x}_t = \boldsymbol{W}_t\boldsymbol{c}_t + \boldsymbol{U}_t\boldsymbol{a}_t + \boldsymbol{U}'_t\boldsymbol{a}'_t + \boldsymbol{V}_t\boldsymbol{b}_t + \boldsymbol{V}'_t\boldsymbol{b}'_t \tag{22}$$

where $\boldsymbol{a}_t \in \mathbb{C}^{(M-N)\times 1}, \boldsymbol{a}'_t \in \mathbb{C}^{N\times 1}$ are the vectors of symbols meant for receiver 1, $\boldsymbol{b}_t \in \mathbb{C}^{(M-N)\times 1}, \boldsymbol{b}'_t \in \mathbb{C}^{N\times 1}$ are those meant for receiver 2, where $\boldsymbol{c}_t \in \mathbb{C}^{M\times M}$ is a common symbol vector, where $\boldsymbol{U}_t = (\hat{\boldsymbol{H}}_t^{(2)})^{\perp} \in \mathbb{C}^{M\times(M-N)}$ is a unit-norm matrix that is orthogonal to $\hat{\boldsymbol{H}}_t^{(2)}$, where $\boldsymbol{V}_t = (\hat{\boldsymbol{H}}_t^{(1)})^{\perp} \in \mathbb{C}^{M\times(M-N)}$ is orthogonal to $\hat{\boldsymbol{H}}_t^{(1)}$, and where $\boldsymbol{W}_t \in \mathbb{C}^{M\times M}, \boldsymbol{U}'_t \in \mathbb{C}^{M\times N}, \boldsymbol{V}'_t \in \mathbb{C}^{M\times N}$ are predetermined randomly-generated matrices known by all nodes.

b) Power and rate allocation: The powers and (normalized) rates during phase s time-slot t, are

$$\begin{aligned}
&P_t^{(\boldsymbol{c})} \doteq P,\ P_t^{(\boldsymbol{a})} \doteq P^{\delta_t^{(2)}} && P_t^{(\boldsymbol{b})} \doteq P^{\delta_t^{(1)}}\\
&P_t^{(\boldsymbol{a}')} \doteq P^{\delta_t^{(2)}-\alpha_t^{(2)}} && P_t^{(\boldsymbol{b}')} \doteq P^{\delta_t^{(1)}-\alpha_t^{(1)}}\\
&r_t^{(\boldsymbol{a})} = (M-N)\delta_t^{(2)} && r_t^{(\boldsymbol{b})} = (M-N)\delta_t^{(1)}\\
&r_t^{(\boldsymbol{a}')} = N(\delta_t^{(2)}-\alpha_t^{(2)})^+ && r_t^{(\boldsymbol{b}')} = N(\delta_t^{(1)}-\alpha_t^{(1)})^+
\end{aligned} \tag{23}$$

where $\{\delta_t^{(1)}, \delta_t^{(2)}\}_{t\in\mathcal{B}_s}$ are designed such that

$$\beta_t^{(i)} \geq \delta_t^{(i)} \quad i = 1, 2,\ t \in \mathcal{B}_s \tag{24}$$

$$\frac{1}{T}\sum_{t\in\mathcal{B}_s}\delta_t^{(1)} = \frac{1}{T}\sum_{t\in\mathcal{B}_s}\delta_t^{(2)} = \bar{\delta} \tag{25}$$

$$\frac{1}{T}\sum_{t\in\mathcal{B}_s}(\delta_t^{(i)} - \alpha_t^{(i)})^+ = (\bar{\delta} - \bar{\alpha}^{(i)})^+ \quad i = 1, 2 \tag{26}$$

for some $\bar{\delta}$ that will be bounded by

$$\bar{\delta} \leq \min\{1, \bar{\beta}^{(1)}, \bar{\beta}^{(2)}, \frac{N(1+\bar{\alpha}^{(1)}+\bar{\alpha}^{(2)})}{M+N}, \frac{N(1+\bar{\alpha}^{(2)})}{M}\} \tag{27}$$

and which will be set to specific values later on, depending on the DoF corner point we wish to achieve.

The exact solutions for $\{\delta_t^{(1)}, \delta_t^{(2)}\}_{t\in\mathcal{B}_s}$ satisfied (24),(25),(26) are shown in [19], and the rates of the common symbols $\{\boldsymbol{c}_{\mathcal{B}_{s,t}}\}_{t=1}^{T}$ are designed to jointly carry

$$T(N - (M-N)\bar{\delta})\log P - o(\log P) \tag{28}$$

bits.

To put the above allocation in perspective, we show the received signals, and describe under each term the order of the summand's average power. These signals take the form

$$\begin{aligned}
\boldsymbol{y}_t^{(1)} = &\underbrace{\boldsymbol{H}_t^{(1)}\boldsymbol{W}_t\boldsymbol{c}_t}_{P} + \underbrace{\boldsymbol{H}_t^{(1)}\boldsymbol{U}_t\boldsymbol{a}_t}_{P^{\delta_t^{(2)}}} + \underbrace{\boldsymbol{H}_t^{(1)}\boldsymbol{U}'_t\boldsymbol{a}'_t}_{P^{\delta_t^{(2)}-\alpha_t^{(2)}}}\\
&+ \underbrace{\boldsymbol{z}_t^{(1)}}_{P^0} + \underbrace{\overbrace{\check{\boldsymbol{H}}_t^{(1)}(\boldsymbol{V}_t\boldsymbol{b}_t + \boldsymbol{V}'_t\boldsymbol{b}'_t)}^{\check{\iota}_t^{(1)}}}_{P^{\delta_t^{(1)}-\alpha_t^{(1)}}} + \underbrace{\overbrace{\ddot{\boldsymbol{H}}_t^{(1)}(\boldsymbol{V}_t\boldsymbol{b}_t + \boldsymbol{V}'_t\boldsymbol{b}'_t)}^{\iota_t^{(1)}-\check{\iota}_t^{(1)}}}_{P^{\delta_t^{(1)}-\beta_t^{(1)}} \leq P^0}
\end{aligned} \tag{29}$$

$$\begin{aligned}
\boldsymbol{y}_t^{(2)} = &\underbrace{\boldsymbol{H}_t^{(2)}\boldsymbol{W}_t\boldsymbol{c}_t}_{P} + \underbrace{\boldsymbol{H}_t^{(2)}\boldsymbol{V}_t\boldsymbol{b}_t}_{P^{\delta_t^{(1)}}} + \underbrace{\boldsymbol{H}_t^{(2)}\boldsymbol{V}'_t\boldsymbol{b}'_t}_{P^{\delta_t^{(1)}-\alpha_t^{(1)}}} + \underbrace{\boldsymbol{z}_t^{(2)}}_{P^0}\\
&+ \underbrace{\overbrace{\check{\boldsymbol{H}}_t^{(2)}(\boldsymbol{U}_t\boldsymbol{a}_t + \boldsymbol{U}'_t\boldsymbol{a}'_t)}^{\check{\iota}_t^{(2)}}}_{P^{\delta_t^{(2)}-\alpha_t^{(2)}}} + \underbrace{\overbrace{\ddot{\boldsymbol{H}}_t^{(2)}(\boldsymbol{U}_t\boldsymbol{a}_t + \boldsymbol{U}'_t\boldsymbol{a}'_t)}^{\iota_t^{(2)}-\check{\iota}_t^{(2)}}}_{P^{\delta_t^{(2)}-\beta_t^{(2)}} \leq P^0}
\end{aligned} \tag{30}$$

where

$$\iota_t^{(1)} \triangleq \boldsymbol{H}_t^{(1)}(\boldsymbol{V}_t\boldsymbol{b}_t + \boldsymbol{V}'_t\boldsymbol{b}'_t),\ \iota_t^{(2)} \triangleq \boldsymbol{H}_t^{(2)}(\boldsymbol{U}_t\boldsymbol{a}_t + \boldsymbol{U}'_t\boldsymbol{a}'_t) \tag{31}$$

denote the interference at receiver 1 and receiver 2 respectively, and where

$$\check{\iota}_t^{(1)} \triangleq \check{\boldsymbol{H}}_t^{(1)}(\boldsymbol{V}_t\boldsymbol{b}_t + \boldsymbol{V}'_t\boldsymbol{b}'_t),\ \check{\iota}_t^{(2)} \triangleq \check{\boldsymbol{H}}_t^{(2)}(\boldsymbol{U}_t\boldsymbol{a}_t + \boldsymbol{U}'_t\boldsymbol{a}'_t) \tag{32}$$

denote the transmitter's delayed estimates of $\iota_t^{(1)}, \iota_t^{(2)}$.

c) Quantizing and broadcasting the accumulated interference: Before the beginning of phase $(s+1)$, the transmitter reconstructs $\check{\iota}_t^{(1)}, \check{\iota}_t^{(2)}$ for all $t \in \mathcal{B}_s$, using its knowledge of delayed CSIT, and quantizes these into

$$\bar{\check{\iota}}_t^{(1)} = \check{\iota}_t^{(1)} - \tilde{\iota}_t^{(1)}, \quad \bar{\check{\iota}}_t^{(2)} = \check{\iota}_t^{(2)} - \tilde{\iota}_t^{(2)} \tag{33}$$

using a total of $N(\delta_t^{(1)} - \alpha_t^{(1)})^+\log P$ and $N(\delta_t^{(2)} - \alpha_t^{(2)})^+\log P$ quantization bits respectively. This allows for bounded power of quantization noise $\tilde{\iota}_t^{(1)}, \tilde{\iota}_t^{(2)}$, i.e, allows for $\mathbb{E}|\tilde{\iota}_t^{(2)}|^2 \doteq \mathbb{E}|\tilde{\iota}_t^{(1)}|^2 \doteq 1$, since $\mathbb{E}|\bar{\iota}_t^{(2)}|^2 \doteq P^{\delta_t^{(2)}-\alpha_t^{(2)}}$, $\mathbb{E}|\bar{\iota}_t^{(1)}|^2 \doteq P^{\delta_t^{(1)}-\alpha_t^{(1)}}$ (cf. [32]). Then the transmitter evenly splits the

$$\sum_{t\in\mathcal{B}_s} \left(N(\delta_t^{(1)} - \alpha_t^{(1)})^+ + N(\delta_t^{(2)} - \alpha_t^{(2)})^+ \right) \log P$$
$$= TN \left((\bar{\delta} - \bar{\alpha}^{(1)})^+ + (\bar{\delta} - \bar{\alpha}^{(2)})^+ \right) \log P \quad (34)$$

(cf. (26)) quantization bits into the common symbols $\{\boldsymbol{c}_t\}_{t\in\mathcal{B}_{s+1}}$ that will be transmitted during the next phase (phase $s+1$), and which will convey these quantization bits together with other new information bits for the receivers. These $\{\boldsymbol{c}_t\}_{t\in\mathcal{B}_{s+1}}$ will help the receivers cancel interference, as well as will serve as extra observations that will allow for decoding of all private information.

Finally, for the last phase S, the main target will be to recover the information on the interference accumulated in phase $(S-1)$. For large S, this last phase can focus entirely on transmitting common symbols.

d) Calibrating the scheme to achieve DoF corner points: The above provided a general description of the structure of the encoding. To achieve the different DoF corner points of Theorem 1, there is a process of calibrating the scheme to achieve these specific DoF corner points. This is not presented here, and - as stated before - is presented in detail in the journal version of this work in [31], which also presents the details of the decoding process.

IV. Conclusions

The work, extending on recent work on the MISO BC, considered the symmetric MIMO BC, and made progress towards establishing and meeting the tradeoff between performance, and feedback timeliness and quality. Considering a general CSIT process, the work provided simple DoF expressions that reveal the role of the number of antennas in establishing the feedback quality associated to a certain DoF performance.

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