MIMO Broadcast and Interference Channels with Location based Partial CSIT

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***Abstract*—Multiple antennas facilitate the coexistence of multiple users in wireless communications, leading to spatial multiplexing and spatial division and to significant system capacity increase. However, this comes at the cost of very precise channel state information at the transmitters (CSIT). We advocate the use of channel propagation models to transform location information into (possibly incomplete) CSIT. We investigate the resulting multi-user sum rate from a DoF (Degree of Freedom, high SNR rate prelog, spatial multiplexing factor) point of view. For single-cell multi-user communications, we argue for a revival of SDMA (Spatial Division Multiple Access). In the MIMO case, the receive antennas can suppress the Non Line of Sight (NLoS) channel components to transform the MIMO channel into a MISO LoS channel, allowing the CSIT to be limited to LoS information. For the multi-cell problem, we consider the feasibility of interference aligment in the case of reduced rank MIMO channels. We then focus on the LoS components. Whereas in general MIMO multi-cell coordinated beamforming, the transmitters require global CSIT due to the coupling between transmit and receive filters, in the LoS case decoupling arises, permitting location based transmit beamforming. We also discuss the transceiver design based on Partial LoS CSIT by essentially maximizing a weighted sum rate at finite SNR and finite Ricean factor.**

I. INTRODUCTION

In TDD single-cell systems, channel reciprocity can turn CSIR (Receiver) into CSIT. In multi-cell systems however, TDD reciprocity is of limited interest as it only leads to only local CSIT. In FDD systems, CSIT needs to be acquired by feedback, which increases with the MIMO, multi-cell and Doppler dimensions. The problem is compounded when taking furthermore user selection into account.

Wireless network based localization offers an alternative and/or complement to GNSS based localization. Satellite connectivity may pose problems in urban canyons and indoor, and not all mobile terminals (MTs) are GNSS equipped. Wireless network based localization is now part of LTE-A, based on the following techniques: Enhanced Cell Id = Cell Id + RSS (Received Signal Strength), O-TDoA (Observed Time Difference of Arrival), and AoA (Angle of Arrival at the base station (BS)).

The availability of location information offers in turn opportunities to enhance the wireless communications. The position based information that can be exploited comprises slow fading channel characteristics of various links:

- LOS/NLOS ((Non) Line of Sight)
- attenuation
- delay spread, frequency selectivity
- angular spreads, MIMO channel characteristics (rank)
- speed, direction of movement, acceleration (predictibility of movement), trajectory

Some of these aspects may require the use of databases (containing these characteristics as a function of position), compatible with a cognitive radio setting. Compared to feedback (FB) based approaches: some of these characteristics can not easily be determined from isolated channel estimates, or not predicted at all (e.g. slow fading prediction). What can not be inferred on the basis of position (as generally believed) is the fast fading state, the instantaneous complex channel impulse response. In what follows, we consider a number of problem formulations in which fast fading state information can essentially be avoided. In this paper, Tx may denote transmit/transmitter/transmission and Rx may denote receive/receiver/reception.

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II. PROPAGATION CHANNEL MODEL

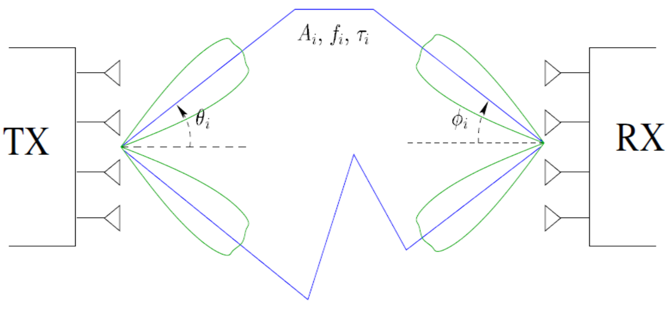

Fig. 1. MIMO transmission with M transmit and N receive antennas.

A. Specular Wireless MIMO Channel Model

Consider a MIMO transmission configuration as depicted in Fig. 1. We get for the matrix impulse response of the time-varying channel $\mathbf{h}(t,\tau)$ [1]

$$\mathbf{h}(t,\tau) = \sum_{i=1}^{N_p} A_i(t)\, e^{j2\pi\, f_i\, t}\, \mathbf{h}_r(\phi_i)\, \mathbf{h}_t^H(\theta_i)\, p(\tau-\tau_i) \quad . \quad (1)$$

The channel impulse response $\mathbf{h}$ has per path a rank 1 contribution in 4 dimensions (Tx and Rx spatial multi-antenna dimensions, delay spread and Doppler spread); there are N_p pathwise contributions where

- A_i: complex attenuation
- f_i: Doppler shift
- θ_i: direction of departure (AoD)
- ϕ_i: direction of arrival (AoA)
- τ_i: path delay (ToA)
- $\mathbf{h}_t^*(.)$, $\mathbf{h}_r(.)$: $M/N \times 1$ Tx/Rx antenna array response
- $p(.)$: pulse shape (Tx filter)

The fast variation of the phase in $e^{j2\pi f_i t}$ and possibly the variation of the A_i (when the nominal path represents in fact a superposition of paths with similar parameters) correspond to the fast fading. All the other parameters (including the Doppler frequency) vary on a slower time scale and correspond to slow fading. We shall assume here OFDM transmission, as is typical for 4G systems, with the Doppler variation over the OFDM symbol duration being negligible. We then get for the channel transfer matrix at any particular subcarrier of a given OFDM symbol

$$\mathbf{H} = \sum_{i=1}^{N_p} A_i \, \mathbf{h}_r(\phi_i) \, \mathbf{h}_t^H(\theta_i) \tag{2}$$

where with some abuse of notation we use the same complex amplitude A_i in which we ignored the dependence on time (particular OFDM symbol), through at least the Doppler shift, and on frequency (subcarrier), through the Tx (and Rx) filter(s).

B. Narrow AoD Aperture (NADA) case

The idea here is to focus on the category of mobiles for which the angular spread seen from the BS is limited [2]. This is a small generalization of the LoS case. In the NADA case, the MIMO channel $\mathbf{H}$ is of the form

$$\mathbf{H} = \sum_i A_i \, \mathbf{h}_r(\phi_i) \mathbf{h}_t^H(\theta_i) \approx \mathbf{B} \, \mathbf{A}^H \ , \quad \mathbf{A} = \left[\mathbf{h}_t(\theta) \; \dot{\mathbf{h}}_t(\theta) \right] \ . \tag{3}$$

In the case of narrow AoD spread, we have

$$\theta_i = \theta + \Delta\theta_i \tag{4}$$

where θ is the nominal (LoS) AoD and $\Delta\theta_i$ is small. Hence

$$\mathbf{h}_t(\theta_i) \approx \mathbf{h}_t(\theta) + \Delta\theta_i \, \dot{\mathbf{h}}_t(\theta) \ . \tag{5}$$

This leads to the second equality in (3). Hence $\mathbf{H}$ is of rank 2 (regardless of the AoA spread). The LOS case is a limiting case in which the power of the $\dot{\mathbf{h}}_t(\theta)$ term becomes negligible and the channel rank becomes 1. The factor $\mathbf{A}$ in $\mathbf{H}$ depends straightforwardly on position (which translates into LOS AoD), only $\mathbf{B}$ (which depends on the $A_i \, \mathbf{h}_r(\phi_i)$ and the $\Delta\theta_i$) remains random.

C. Partial CSIT LoS Channel Model

Assuming the Tx disposes of not much more than the LoS component information, we shall consider the following MIMO channel model

$$\mathbf{H} = \mathbf{h}_r \, \mathbf{h}_t^H(\theta) + \widetilde{\mathbf{H}} \tag{6}$$

where θ is the LoS AoD and the Tx side array response is normalized: $||\mathbf{h}_t(\theta)||^2 = 1$. Since the orientation of the MT is random, and the LoS case can be considered as a limiting NADA case in which a multitude of AoAs could appear, we shall model the Rx side LoS array response $\mathbf{h}_r$ as a vector of i.i.d. complex Gaussian variables

$$\begin{array}{ll} \mathbf{h}_r \ \text{i.i.d.} \ \sim \mathcal{CN}(0, \frac{\mu}{\mu+1}) & \text{and} \\ \widetilde{\mathbf{H}} \ \text{i.i.d.} \ \sim \mathcal{CN}(0, \frac{1}{\mu+1}\frac{1}{M}) & \text{, independent of } \mathbf{h}_r , \end{array} \tag{7}$$

where the matrix $\widetilde{\mathbf{H}}$ represents the aggregate NLoS components. Note that

$$\begin{array}{l} \mathrm{E}||\mathbf{H}||_F^2 = \mathrm{Etr}\{\mathbf{H}^H\mathbf{H}\} = \\ ||\mathbf{h}_t(\theta)||^2 \ \mathrm{E}||\mathbf{h}_r||^2 + \mathrm{E}||\mathbf{H}||_F^2 = \frac{\mu N}{\mu+1} + \frac{N}{\mu+1} = N \, , \end{array} \tag{8}$$

reflecting that Rx power augments proportionally with N, and $(\mathrm{E}||\mathbf{h}_r \, \mathbf{h}_t^T(\theta)||_F^2)/(\mathrm{E}||\mathbf{H}||_F^2) = \mu$ which can be considered as a Rice factor. In fact the only parameter additional to the LoS AoD θ assumed in (6) is μ.

III. Single-Cell Multi-User Communications

In this section, we shall focus on the downlink (DL) of Spatial Division Multiple Access (SDMA) problem, which in Information Theory is called the Broadcast Channel (BC). The SDMA terminology dates from the early nineties. These days it is referred to as the multi-user MISO (or MIMO) communications problem.

A. Location Based SDMA

The MISO case was treated in [3], where we proposed that location based MU MIMO transmission involve position based user selection (attenuation, nominal AoD, AoD spread) and associated beamforming (BF) and power control (PC). For the location aided MISO case, we need to essentially consider users with LoS channels. The effect of location error or of weak multipath on the resulting sum rate was also investigated.

In the MIMO BC case, the multiple Rx antennas at the MTs cannot help to augment the totale number of streams, which are limited by the number of BS antennas. They allow to vary the number of streams per user though and thus to combine SU MIMO with MU MIMO. In the case of partial CSIT (as in LTE-A), it may seem beneficial to augment the number of streams per user. This is because in comparison the CSIR can be considered as as good as perfect (semi-blind channel estimation can be performed at the Rx). This means that at least the interference due to CSIT imperfections coming from other streams of the same user can be handled by the Rx, in effect reducing the number of undecoded interfering streams. Ignoring CSIT imperfections, on the basis of diversity considerations it may be beneficial to have some MIMO aspect per user (to distribute the Zero-Forcing (ZF) or Minimum Mean Squared Error (MMSE) task between Tx and Rx) in case of a rich channel model [4], whereas in the perhaps more realistic case of poorer multipath a single stream per user is to be preferred [5]. The effect of user selection may play a role also. In the context of location aided, we shall consider good but incomplete CSIT. As a result, it may be beneficial to focus on the case of a single stream per user.

B. MIMO BC with Incomplete CSIT

The concept of incomplete CSIT was considered in [6] for the MIMO IC with a single stream per link. What is shown in [6] is that in the MIMO IC case, ZF may be possible with less than global CSIT. However, this only occurs for cases of very non-uniform antenna numbers over the Txs and Rxs. Incomplete CSIT differs from partial CSIT, which usually means that the Tx has a noisy (possibly quantized) version of the channel. In incomplete CSIT, the knowledge is (close to) perfect, but only of part of the channel (with the rest being unknown).

Consider MIMO BC with a single stream per user. We shall show that it is sufficient for ZF BF (and hence for DoF) that the BS knows for each user a vector in the row span of its MIMO channel. Let $\mathbf{H}_k$ be the $N \times M$ MIMO channel of user k and let $\mathbf{c}_k\mathbf{H}_k$ be the equivalent MISO channel that the BS is aware of for user k, where $\mathbf{c}_k$ is a $1 \times N$ vector (the number of receive antennas can vary with user but we shall keep the notation as N). For ZF BF, the BS shall use for user k a spatial filter $\mathbf{g}_k$ such that

$$\mathbf{g}_k^H = \mathbf{c}_k\mathbf{H}_k P^{\perp}_{(\mathbf{cH})_{\overline{k}}} / ||\mathbf{c}_k\mathbf{H}_k P^{\perp}_{(\mathbf{cH})_{\overline{k}}}|| \tag{9}$$

where $P^{\perp}_{\mathbf{X}} = I - P_{\mathbf{X}}$ and $P_{\mathbf{X}} = \mathbf{X}^H(\mathbf{X}\mathbf{X}^H)^{-1}\mathbf{X}$ are projection matrices and $(\mathbf{cH})_{\overline{k}}$ denotes the stacking of $\mathbf{c}_i\mathbf{H}_i$ for users $i = 1, \ldots, M$, $i \neq k$. The $N \times 1$ received signal at user k is

$$\mathbf{y}_k = \mathbf{H}_k\,\mathbf{g}_k\,x_k + \sum_{i=1,\neq k}^{M} \mathbf{H}_k\,\mathbf{g}_i\,x_i + \mathbf{v}_k \tag{10}$$

where x_i is the signal intended for user i, and $\mathbf{v}_k$ is additive noise. For the reception, user k shall use a linear filter $\mathbf{f}_k$ with corresponding Signal-to-Interference Ratio (SIR) (for ZF and DoF considerations)

$$\mathrm{SIR}_k = \frac{|\mathbf{f}_k\mathbf{H}_k\mathbf{g}_k|^2\sigma_k^2}{\sum_{i=1,\neq k}^{M}|\mathbf{f}_k\mathbf{H}_k\mathbf{g}_i|^2\sigma_i^2} \tag{11}$$

where σ_i^2 is the power of x_i. Maximizing SIR_k w.r.t. $\mathbf{f}_k$ leads to $\mathbf{f}_k \sim \mathbf{c}_k$ if $\mathbf{H}_k$ is full row rank (and to $\mathbf{f}_k\mathbf{H}_k \sim \mathbf{c}_k\mathbf{H}_k$ otherwise) with maximum SIR

$$\max_{\mathbf{f}_k} \mathrm{SIR}_k = \mathrm{SIR}_k(\mathbf{f}_k = \mathbf{c}_k) = \infty \tag{12}$$

since the interference power becomes zero. This leads to:

Theorem 1: **Sufficiency of Incomplete CSIT for Full DoF in MIMO BC** In the MIMO BC with perfect CSIR, it is sufficient that the BS knows for each of the K users any vector in the row space of its MIMO channel (as long as the resulting vectors are linearly independent) in order for ZF BF to produce $\min(M, K)$ DoF.

C. Location Based MU-MIMO

Consider now a propagation channel model as in (2), combined with the NADA model, in which a cluster of paths contributes the equivalent of two paths. Counting paths in this way, assume that the total number ${N_p}'$ of equivalent paths satisfies ${N_p}' \leq N$ (again, the number of Rx antennas could be variable with user but we shall omit this notation).

Assume one of the paths is the LoS path which is known by the BS (location aided). The situation that arises now is similar to chip equalization in the 3G downlink: due to the synchronicity of the downlink, 3G systems use CDMA with orthogonal codes such that a simple correlator at the Rx would suffice to suppress all intracell interference. This ideal scenario is perturbed by the multipath propagation channel whose frequency-selectivity perturbs the orthogonality of the spreading codes. However, since in the downlink from the BS to a particular user all intracell signals pass through the same channel of that user, it suffices for the user to equalize that channel to restore the code orthogonality and to allow a correlator to suppress the interference. In SDMA, the temporal spreading of CDMA is replaced by spatial filtering at the BS. This spatial filtering $\mathbf{g}_k$ is based on the hypothesis of a LoS channel. Hence, for the reception at the user through the LoS path, all interference will be supressed. But the interference arrives at the user through the multipath components. However, regardless of the beamforming employed at the BS, all interference received by user k passes through the same multipath components of the channel of that user. Now, if ${N_p}' \leq N$, the user can employ Rx spatial filtering $\mathbf{f}_k$ to suppress all paths, except for the LoS path, so that $\mathbf{f}_k\mathbf{H}_k$ only contains the LoS path. Combined with the LoS based BF $\mathbf{g}_k$ design, this allows to suppress all interference and hence to produce full DoF.

For the previous reasoning to work, it would have been sufficient (in terms of DoF) that the Bs knows any vector in the row space of $\mathbf{H}_k$, but clearly the LoS path is typically much stronger than the other paths. Hence the knowledge of the LoS path leads to better performance at finite SNR. When there is no LoS path, it suffices to use another path, preferably the strongest path. Whereas the LoS path can be computed on the basis of only the user position (and a calibrated antenna array), in case another (and hence single- or multi-bounce) path needs to be used, this will typically require a database containing the information of the AoD of the strongest path, as a function of the position of the user.

When ${N_p}' > N$, there is no guarantee that the LoS Tx antenna array response lies in the row space of the MIMO channel matrix.

D. Location Based MU-MIMO: from ZF to BF design at finite SNR and Rice factor

In the previous subsection we considered the attainable DoF with LoS CSIT, attained by ZF Tx BF design on the LoS components. Note in passing that in practical multipath scenarios, even if only the interference passing through the LoS paths would be handled, this would already lead to a substantial SINR increase. Here we shall explore how to go beyond the asymptotics of high SNR and high Ricean factor: even if the Tx ignores the multipath and the Rx can handle it, it would be better to have a multipath aware Tx design. To this end, various intermediate forms of CSIT could be considered beyond the LoS knowledge only. Here we shall consider the perhaps simplest model, the partial CSIT LoS model of (6). Note that the Ricean factor μ satisfies uplink/downlink (UL/DL) reciprocity, even in a FDD system, and hence can easily be estimated.

D.1. Perfect CSI Case

So consider the signal Rx'd by user k in (10) where we shall now assume that the $K \leq M$ signal streams x_i have unit variance and the noise is white with $\mathbf{v}_k \sim \mathcal{CN}(0, \sigma^2_{v,k} I_{N_k})$ (for N_k Rx antennas). Consider as a starting point for the optimization the weighted sum rate (WSR)

$$WSR = WSR(\mathbf{g}) = \sum_{k=1}^{K} u_k \ \ln \frac{1}{e_k} \quad (13)$$

where $\mathbf{g}$ represents the collection of BFs $\mathbf{g}_k$, the u_k are rate weights, the $e_k = e_k(\mathbf{g})$ are the Minimum Mean Squared Errors (MMSEs)

$$\begin{aligned} \frac{1}{e_k} &= 1 + \mathbf{g}_k^H \mathbf{H}_k^H \mathbf{R}_{\overline{k}}^{-1} \mathbf{H}_k \mathbf{g}_k = (1 - \mathbf{g}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{g}_k)^{-1} \\ \mathbf{R}_k &= \mathbf{R}_{\overline{k}} + \mathbf{H}_k \mathbf{g}_k \mathbf{g}_k^H \mathbf{H}_k^H \\ \mathbf{R}_{\overline{k}} &= \textstyle\sum_{i \neq k} \mathbf{H}_k \mathbf{g}_i \mathbf{g}_i^H \mathbf{H}_k^H + \sigma^2_{v,k} I_{N_k}) \ , \end{aligned} \quad (14)$$

$\mathbf{R}_k, \mathbf{R}_{\overline{k}}$ are the total and interference plus noise Rx covariance matrices resp. and e_k is the MMSE obtained at the output $\widehat{x}_k = \mathbf{f}_k^H \mathbf{y}_k$ of the optimal (MMSE) linear Rx $\mathbf{f}_k$,

$$\mathbf{f}_k = \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{g}_k \ . \quad (15)$$

For a general Rx filter $\mathbf{f}_k$ we have the MSE

$$\begin{aligned} e_k(\mathbf{f}_k, \mathbf{g}) &= (1 - \mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_k)(1 - \mathbf{g}_k^H \mathbf{H}_k^H \mathbf{f}_k) \\ &\quad + \textstyle\sum_{i \neq k} \mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_i \mathbf{g}_i^H \mathbf{H}_k^H \mathbf{f}_k + \sigma^2_{v,k} ||\mathbf{f}_k||^2 = \\ 1 - &\mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_k - \mathbf{g}_k^H \mathbf{H}_k^H \mathbf{f}_k + \textstyle\sum_i \mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_i \mathbf{g}_i^H \mathbf{H}_k^H \mathbf{f}_k + \sigma^2_{v,k} ||\mathbf{f}_k||^2 . \end{aligned} \quad (16)$$

The $WSR(\mathbf{g})$ is a non-convex and complicated function of $\mathbf{g}$. In [7], [8], we introduced an augmented cost function, the Weighted Sum MSE, $WSMSE(\mathbf{g}, \mathbf{f}, w)$

$$= \sum_{k=1}^{K} u_k (w_k \ e_k(\mathbf{f}_k, \mathbf{g}) - \ln w_k) + \lambda (\sum_{k=1}^{K} ||\mathbf{g}_k||^2 - P) \quad (17)$$

where λ is a Lagrange multiplier and P is the Tx power constraint. After optimizing over the aggregate auxiliary Rx filters $\mathbf{f}$ and weights w, we get the WSR back:

$$\min_{\mathbf{f}, w} WSMSE(\mathbf{g}, \mathbf{f}, w) = -WSR(\mathbf{g}) \ . \quad (18)$$

The advantage of the augmented cost function is however that alternating optimization for one of the three sets of quantities, $\mathbf{g}, \mathbf{f}, w$, keeping the other two fixed, leads to solving simple quadratic or convex functions

$$\begin{aligned} &\min_{w_k} WSMSE \ \Rightarrow \ w_k = 1/e_k \\ &\min_{\mathbf{f}_k} WSMSE \ \Rightarrow \ \mathbf{f}_k = (\sum_i \mathbf{H}_k \mathbf{g}_i \mathbf{g}_i^H \mathbf{H}_k^H + \sigma^2_{v,k} I_{N_k})^{-1} \mathbf{H}_k \mathbf{g}_k \\ &\min_{\mathbf{g}_k} WSMSE \ \Rightarrow \\ &\qquad \mathbf{g}_k = (\textstyle\sum_i u_i w_i \mathbf{H}_i^H \mathbf{f}_i \mathbf{f}_i^H \mathbf{H}_i + \lambda I_M)^{-1} \mathbf{H}_k^H \mathbf{f}_k u_k w_k \end{aligned} \quad (19)$$

Indeed, after substituting (16) into (17), one can notice the UL/DL duality, leading to a duality between Tx and Rx filters and the optimal Tx filter g_k in (19) is indeed of the form of a MMSE linear Rx for the dual UL in which λ plays the role of Rx noise variance. The optimal λ in each iteration can be found using a (bisection) line search on $\sum_{k=1}^{K} ||\mathbf{g}_k||^2 - P = 0$ or as in [7], [8] by exploiting $\sum_k \mathbf{g}_k^H \frac{\partial WSMSE}{\partial \mathbf{g}_k^*} = 0$.

D.2. Partial CSIT Case

Now, so far we have assumed that the channel $\mathbf{H}$ is known. The scenario of interest however is that of perfect CSIR but partial LoS CSIT. Once the CSIT is imperfect, various optimization criteria could be considered, such as outage capacity. Here we shall consider the ergodic weighted sum rate $\mathrm{E}_{\mathbf{H}} WSR(\mathbf{g}, \mathbf{H}) =$

$$EWSR(\mathbf{g}) = \ \mathrm{E}_{\mathbf{H}} \sum_k u_k \ln(1 + \mathbf{g}_k^H \mathbf{H}_k^H \mathbf{R}_{\overline{k}}^{-1} \mathbf{H}_k \mathbf{g}_k) \quad (20)$$

where we now underlign the dependence of various quantities on $\mathbf{H}$. The EWSR in (20) corresponds to perfect CSIR since the optimal Rx filters $\mathbf{f}_k$ as a function of the aggregate $\mathbf{H}$ have been substituted, namely $WSR(\mathbf{g}, \mathbf{H}) = \max_{\mathbf{f}} \sum_k u_k (-\ln(e_k(\mathbf{f}_k, \mathbf{g})))$. However, $EWSR(\mathbf{g})$ is again difficult to maximize directly. As observed in [9], it appears much more attractive to consider $\mathrm{E}_{\mathbf{H}} e_k(\mathbf{f}_k, \mathbf{g}, \mathbf{H})$ since $e_k(\mathbf{f}_k, \mathbf{g}, \mathbf{H})$ is quadratic in $\mathbf{H}$. Hence in [9], the cost function optimized is $\mathrm{E}_{\mathbf{H}} WSMSE(\mathbf{g}, \mathbf{f}, w, \mathbf{H})$ where $WSMSE(\mathbf{g}, \mathbf{f}, w, \mathbf{H})$ appears in (17). However,

$$\begin{aligned} &\min\nolimits_{\mathbf{f}, w} \ \mathrm{E}_{\mathbf{H}} WSMSE(\mathbf{g}, \mathbf{f}, w, \mathbf{H}) \\ &\geq \ \mathrm{E}_{\mathbf{H}} \min\nolimits_{\mathbf{f}, w} WSMSE(\mathbf{g}, \mathbf{f}, w, \mathbf{H}) = -EWSR(\mathbf{g}) \, . \end{aligned} \quad (21)$$

So now only a lower bound to the EWSR gets maximized, which corresponds in fact to the CSIR being equally partial as the CSIT. The EWSR gap can be reduced by following the optimization over the Tx filters $\mathbf{g}_k$ with an optimization over the Rx filters $\mathbf{f}_k$ for full CSIR, namely by taking the $\mathbf{f}_k$ as in (15).

D.3. Partial LoS CSIT Case

Consider now the partial LoS CSIT case (6), namely $\mathbf{H}_k = \mathbf{h}_{r,k} \, \mathbf{h}_{t,k}^H + \widetilde{\mathbf{H}}_k$ and introduce the variances $\sigma_k^2 = \frac{\mu_k}{\mu_k + 1}$ for the elements of $\mathbf{h}_{r,k}$ and $\widetilde{\sigma}_k^2 = \frac{1}{\mu_k + 1} \frac{1}{M}$ for the elements of $\widetilde{\mathbf{H}}_k$ (which may also reflect link attenuations).

However, the problem in applying this approach to the partial LoS CSIT case (6) is that $\mathrm{E}\mathbf{H}_k = 0$ and hence the first order terms $\mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_k$ in the last expression of (16) disappear, making the minimization of $\mathrm{E}_{\mathbf{H}}e_k(\mathbf{f}_k,\mathbf{g},\mathbf{H})$ meaningless. A solution to this problem can be find by reexamining the (fictitious) dual Rx problem to which the BF $\mathbf{g}_k$ was the optimal Rx solution:

$$\begin{aligned}\mathbf{g}_k^H\widetilde{y}_k &= \mathbf{g}_k^H\mathbf{H}_k^H\mathbf{f}_k\,\widetilde{x}_k + \mathbf{g}_k^H(\textstyle\sum_{i\neq k}\mathbf{H}_i^H\mathbf{f}_i\,\widetilde{x}_i + \widetilde{v}_k)\\ &= \mathbf{g}_k^H\mathbf{h}_{t,k}\underbrace{\mathbf{h}_{r,k}^H\mathbf{f}_k\,\widetilde{x}_k}_{\text{source } z_k} + \mathbf{g}_k^H\underbrace{(\widetilde{\mathbf{H}}_k^H\mathbf{f}_k\,\widetilde{x}_k + \sum_{i\neq k}\mathbf{H}_i^H\mathbf{f}_i\,\widetilde{x}_i +}_{\text{interference}}\underbrace{\widetilde{v}_k}_{\text{noise}})\end{aligned} \tag{22}$$

where the fictitious signals $\widetilde{x}_i$ have variance $u_i w_i$, $\mathrm{E}\widetilde{v}_k\widetilde{v}_k^H = \lambda I_M$, we integrate the random $\mathbf{h}_{r,k}^H$ into the source z_k which now has variance $\sigma_k^2||\mathbf{f}_k||^2$, and the multipath part $\widetilde{\mathbf{H}}_k^H\mathbf{f}_k\,\widetilde{x}_k$ is counted in the interference. This dual Rx problem leads to the SINR

$$\begin{aligned}\widetilde{\mathrm{SINR}}_k &= \frac{|\mathbf{g}_k^H\mathbf{h}_{t,k}|^2\sigma_k^2\alpha_k}{\mathbf{g}_k^H\widetilde{\mathbf{R}}_k\mathbf{g}_k}\quad,\quad \alpha_i = u_i w_i||\mathbf{f}_i||^2\\ \widetilde{\mathbf{R}}_k &= (\lambda + \sum_{i=1}^{K}\widetilde{\sigma}_i^2\alpha_i)I_M + \sum_{i\neq k}\sigma_i^2\alpha_i\mathbf{h}_{t,i}\mathbf{h}_{t,i}^H\end{aligned} \tag{23}$$

with maximizing solution

$$\mathbf{g}_k = \widetilde{\mathbf{R}}_k^{-1}\mathbf{h}_{t,k}\;. \tag{24}$$

This Tx filter solution only depends on the Rx filters through the $||\mathbf{f}_k||^2$. In order to determine these (approximately), consider the MSE in a first instance only averaged over the $\widetilde{\mathbf{H}}$, $\min_{\mathbf{f}_k}\mathrm{E}_{\widetilde{\mathbf{H}}}e_k(\mathbf{f}_k,\mathbf{g},\mathbf{H})$ (where $\mathrm{E}_{\widetilde{\mathbf{H}}}\{.\} = \mathrm{E}_{\mathbf{H}|\mathbf{h}_r}\{.\}$), leading to

$$\mathbf{f}_k = \left(\sigma_{v,k}^2 I + \sum_{i=1}^{K}\{|\mathbf{h}_{t,i}^H\mathbf{g}_i|^2\mathbf{h}_{r,i}\mathbf{h}_{r,i}^H + \widetilde{\sigma}_i^2||\mathbf{g}_i||^2 I\right)^{-1}\mathbf{h}_{r,k}\mathbf{h}_{t,k}^H\mathbf{g}_k \tag{25}$$

We then can obtain

$$\begin{aligned}||\mathbf{f}_k||^2 &\approx \mathrm{E}_{\mathbf{h}_r}||\mathbf{f}_k||^2 \approx |\mathbf{h}_{t,k}^H\mathbf{g}_k|^2\sigma_k^2\ \mathrm{E}_{\mathbf{h}_r}\mathrm{tr}\{(.)^{-2}\}\\ &\approx |\mathbf{h}_{t,k}^H\mathbf{g}_k|^2\sigma_k^2\ \mathrm{tr}\{(\mathrm{E}_{\mathbf{h}_r}.)^{-2}\}\\ &= \frac{\sigma_k^2|\mathbf{h}_{t,k}^H\mathbf{g}_k|^2 N_k}{(\sigma_{v,k}^2 + \sum_i(\sigma_i^2|\mathbf{h}_{t,i}^H\mathbf{g}_i|^2 + \widetilde{\sigma}_i^2||\mathbf{g}_i||^2))^2}\end{aligned} \tag{26}$$

and similarly using (19), (14)

$$w_k \approx \left(1 - \frac{\sigma_k^2|\mathbf{h}_{t,k}^H\mathbf{g}_k|^2 N_k}{\sigma_{v,k}^2 + \sum_i(\sigma_i^2|\mathbf{h}_{t,i}^H\mathbf{g}_i|^2 + \widetilde{\sigma}_i^2||\mathbf{g}_i||^2)}\right)^{-1} \tag{27}$$

which depend on the $\mathbf{g}_k$ through $||\mathbf{g}_k||^2$ and $|\mathbf{h}_{t,i}^H\mathbf{g}_i|^2$. The solution for the Tx filters $\mathbf{g}_k$ requires alternating between solving for the $\mathbf{g}_k$ from (24), (23), with adjustment of the Lagrange multiplier λ by line search for $\sum_{k=1}^{K}||\mathbf{g}_k||^2 - P = 0$, and solving for the $||\mathbf{f}_k||^2$, w_k from (26), (27).

In the case of high Ricean factor and high SNR, $\widetilde{\mathbf{R}}_k$ in the denominator of $\widetilde{\mathrm{SINR}}_k$ is dominated by the term $\sum_{i\neq k}\sigma_i^2||\mathbf{f}_i||^2\mathbf{h}_{t,i}\mathbf{h}_{t,i}^H$ and hence the $\widetilde{\mathrm{SINR}}_k$ gets maximized by enforcing $\mathbf{h}_{t,i}^H\mathbf{g}_k = 0$, $i \neq k$ which is the ZF of the LoS components considered earlier.

Again, in the end the Rxs use the full CSIR solutions (15).

IV. MIMO Interference Channel (IC)

A. IA feasibility singular MIMO IC

This subject was treated by [10] for the general $K = 2$ user case and for certain symmetric cases with $K = 3$. Related work also appears in [11] where for the case of no relay (as considered here) only some bounds were provided.

Interference Alignment (IA) is joint Tx/Rx ZF BF and allows to attain the correct DoF in an IC. For d_k streams of user k, a $M_k \times d_k$ Tx filter $\mathbf{G}_k$ and a $d_k \times N_k$ Rx filter $\mathbf{F}_k$ is used. In the rank deficient case, if $0 \leq r_{ik} \leq \min(N_i, M_k)$ denotes the rank of MIMO channel $\mathbf{H}_{ik}$ then we can factor $\mathbf{H}_{ik} = \mathbf{B}_{ik}\mathbf{A}_{ik}$ for some full rank $N_i \times r_{ik}$ $\mathbf{B}_{ik}$ and $r_{ik} \times M_k$ $\mathbf{A}_{ik}$. The ZF from BS k to MT i requires

$$\mathbf{F}_i\mathbf{H}_{ik}\mathbf{G}_k = \mathbf{F}_i\mathbf{B}_{ik}\mathbf{A}_{ik}\mathbf{G}_k = 0 \tag{28}$$

which involves $\min(d_i d_k, d_i r_{ik}, r_{ik} d_k)$ constraints to be satisfied by the $(N_i - d_i)d_i/(M_k - d_k)d_k$ variables parameterizing the row/column subspaces of $\mathbf{F}_i/\mathbf{G}_k$. The overall IA feasibility gets determined by verifying whether the system is proper [6]: for each subset of MTs and subset of BSs, the total number of Tx/Rx variables involved needs to be at least equal to the total number of constraints in the corresponding conditions (28). When the rank constraints are active (number of constraints involves r_{ik}), counting the # of variables vs. the # of ZF constraints gives the complete answer since we have traditional (one-sided) Tx or Rx ZF ($\mathbf{F}_i\mathbf{B}_{ik} = 0$ or $\mathbf{A}_{ik}\mathbf{G}_k = 0$). When the rank constraints are not active (min attained for $d_i d_k$) then counting arguments may not be sufficient in very rectangular (non-square) MIMO channel cases [12], [13]. Note also that the full rank requirement on $\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k$ leads to $1 \leq d_k \leq r_{kk} \leq \min(N_k, M_k)$ (the first inequality reflects that we consider only active links).

Consider now some examples that were also considered in [10]. In the full rank case with K links of $N \times M$, we have that $d_k = \frac{M+N}{K+1}$ is feasible in the not too rectangular case. In the uniform singular $K = 2$ case with $(M, N, r)^2$, $d = \min(r, \dfrac{M+N-r}{2})$ is possible (with $d_1 = d_2 = d$). For the uniform square $K = 3$ case $(M, M, r)^3$, $d = \min(r, M/2)$ is feasible. Still in the $K = 3$, $M \times M$ case with

$$r_{ik} = \begin{cases} r_0 & , i = k\\ r_1 & , i > k\\ r_2 & , i < k\end{cases} \tag{29}$$

we get $d = \min(r_0, M - \dfrac{\min(M, r_1 + r_2)}{2})$.

B. IA feasibility singular MIMO IC with Tx/Rx decoupling

In this case we shall insist that (28) be satisfied by

$$\mathbf{F}_i\mathbf{B}_{ik} = 0 \text{ or } \mathbf{A}_{ik}\mathbf{G}_k = 0\;. \tag{30}$$

This leads to a possibly increased number of ZF constraints $r_{ik}\min(d_i, d_k)$ and hence to possibly reduced IA feasibility. Of course, the task of ZF of every cross link now needs to be partitioned between all Txs and Rxs, taking into account the limited number of variables each Tx or Rx has. The main goal of this approach however is that it leads to Tx/Rx decoupling. Whereas in the general case (28) the design of the Txs depends on the Rxs and vice versa, in (30) the ZF constraints are linear and involve Tx or Rx but not both. The ZF constraints for a Tx (or a Rx) only require local channel knowledge (of the channel connected to it). Of course, the gobal ZF task partitioning needs to be known. This leads to a category of IA feasibility with incomplete CSIT, different from the one appearing in [6] as described earlier.

In the uniform case $(M, N, r)^K$ with $d \leq r$ per user, (30) leads to

$$d \leq \frac{1}{2}(M + N - (K-1)r) \tag{31}$$

whereas the general coupled case (28) would have led to $d \leq \frac{1}{2}(M + N - (K-1)d)$. There is no loss if $d = r$, in which case $d = r \leq \frac{M+N}{K+1}$.

In the case of general rank distribution but with a single stream per user ($d_k \equiv 1$), we get

$$\sum_{i=1}^{K}(M_k + N_k) \geq 2K + \sum_{i \neq k} r_{ik} \ . \tag{32}$$

The non-decoupled case would correspond to replacing all the r_{ik} in (32) by 1.

C. LoS Case

In what follows, we shall focus on the LoS limit for considerations of location based processing. This is a special case of (31) with $d = r = 1$ and leads to the requirement

$$M + N \geq K + 1 \tag{33}$$

for IA feasibility with a single stream per user. In the MISO or SIMO cases this becomes $M \geq K$ or $N \geq K$. The meaning of (33) is: $(M-1)+(N-1) \geq K-1$: that each BS performs ZF towards $M-1$ MTs. As a result, each MT still receives interference from $(K-1)-(M-1)$ cross links but with its N antennas it can ZF $N-1$ streams.

In the decoupled approach, the design of any Tx $\mathbf{g}_k$ only depends on the factors A_{ik} of the channels connected to it and in general even only on a subset of this local CSIT (e.g. in the LoS case, only $M-1$ cross link A_{ik} are required to be known for any given BS). In the LoS case, the A_{ik} are clearly only a function of the positions of the BS and MTs (and the BS antenna array response).

One can go somewhat beyond the LoS case by considering (LoS) NADA and other multipath components. The number of components (and hence the r_{ik}) to be considered could vary with the cross links. An issue that arises here is that different cross links may have multipath components with the same AoD from a certain BS, because the paths may bounce on the same scatterer. In this case the multiple paths get ZF'd simultaneously, leading to a reduction in required Tx antennas.

D. BF Design in the Finite SNR and Rice Factor Case with Partial LoS CSIT

In the case of a centralized design, the approach of subsection III.D.3 can be followed with a few minor modifications: the channels $\mathbf{H}_{ik}$ now have two indices, and now there is a Tx power constraint per BS leading to $||\mathbf{g}_k||^2 = P_k$ (with associated Lagrange multiplier λ_k). As a result the LoS and NLoS variances σ_{ik}^2 and $\widetilde{\sigma}_{ik}^2$ now also have two indices, and they should not only reflect the Ricean factors μ_{ik} but also link attenuations.

Consider now the case of a decoupled Tx/Rx design, which should be based on Local Partial LoS CSIT only. In the dual UL (22), the LoS components of only $M-1$ cross links should be accounted for, namely the ones that the particular BS considered is supposed to reduce its interference over. For the $K-M$ other cross links, the LoS component can be handled like a NLoS component, with the σ_{ik}^2 absorbed in the corresponding $\widetilde{\sigma}_{ik}^2$. This means that the number of terms in the last sum for $\widetilde{\mathbf{R}}$ in (23) shrinks from $K-1$ to $M-1$. For the determination of the $||\mathbf{f}_k||^2$, w_k from (26), (27), a number of assumptions will need to be made on the values for the P_i, σ_{ik}^2, $\widetilde{\sigma}_{ik}^2$ and $\sigma_{v,k}^2$ involved. In any case, we can take for sure $||\mathbf{g}_i||^2 = P_i$ and assuming an isotropic distribution of the $\mathbf{g}_i$ compared to the $\mathbf{h}_{t,i}$, we can set $|\mathbf{h}_{t,i}^H \mathbf{g}_i|^2 = P_i/M$.

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