MIMO Frequency-Selective Channel Modeling based on Pathwise Dynamics

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***Abstract*— A specular approach to model MIMO frequency-selective channel variations, that yields a parsimonious channel representation, is proposed. The specular structure is shown to simplify linear estimation and prediction of the channel. Identifiability of specular channels is studied, and an algorithm achieving identification is proposed. The performance of the proposed method is evaluated through computer simulations.**

I. Introduction

The use of specular models for channel analysis and tracking has been proposed by various authors seeking to improve the ability to accurately estimate [1], represent and transmit [2], or predict [3], [4] Channel State Information (CSI). Specular methods constitue viable candidates for channel tracking and prediction, since the insight they provide into the actual channel structure – namely, separation of the channel variation into its space and time components – can improve the performance and decrease the complexity of channel tracking and prediction. Various methods have been proposed to estimate the underlying parameters, including MUSIC in [3], ESPRIT in [4] and SAGE in [1].

In the present contribution, we propose to use a specular (pathwise) approach in order to gain access to a reduced parameter set representing the channel state, in order to improve channel estimation (smoothing) and prediction. After recalling the specular channel model in section II, and outlining in section III how it makes channel estimation and prediction easier, we provide sufficient conditions for identifiability of a specular channel in section IV, and an algorithm, based on simultaneous diagonalization of the covariance matrices, that achieves identification is proposed in section V. Section VII presents simulation results.

II. Specular channel model

Let us consider a Multiple-Input Multiple-Output (MIMO) frequency-selective channel, with N_t transmit (Tx) and N_r receive (Rx) antennas. The impulse response of the channel between the i^{th} Tx antenna and the j^{th} Rx antenna is denoted by $h_{i,j}(t,\tau)$, where t is the time and τ is the lag.

We will henceforth work under the assumption that the channel state evolves according to a specular model. In such a model, each impulse response $h_{i,j}(t,\cdot)$ is the superposition of a finite number P of discrete paths at lag $\tau_p^{(i,j)} = l_p^{(i,j)} T_s$, $p = 1 \dots P$, resulting from either line-of-sight propagation, or one or several reflections. This model relies upon the fact that the paths between all the Tx-Rx antenna pairs have most of their characteristics in common, except for what happens near the antenna arrays. Hence, they share some properties, namely their speed w.r.t. the reflectors, and the reflection characteristics (hence their Doppler and gain are the same whatever antenna pair is considered). Each path coefficient can be decomposed into a product of two components:

- a space component $\alpha_p^{(i,j)}$, which depends on the physical properties of path p between Tx antenna i and Rx antenna j, including antennas and reflectors position, path loss, etc.
- a time component $\beta_p(t)$ which includes the Doppler due to reflectors motion and the relative speed of the transmitter w.r.t. the receiver.

The time components $\beta_p(t)$ are assumed to be independent between paths, hence $p \neq p' \Rightarrow \mathcal{E}_n[\beta_p(t)\,\beta_{p'}(t')] = 0 \quad \forall (t,t')$. Note that we consider a time scale where they evolve significantly during time, *e.g.* due to the Doppler effect, and hence can be considered random processes, whereas the physical properties of the problem, comprised of the $\alpha_p^{(i,j)}$, do not vary.

In discrete time, the specular channel model yields

$$h_{n,l}^{(i,j)} \triangleq h_{i,j}(nT_s, lT_s) = \sum_{p=1}^{P^{(i,j)}} \alpha_p^{(i,j)} \beta_{n,p} \delta_{l_p^{(i,j)}}(l)\,, \quad (1)$$

where we used the discretized version of the time component $\beta_{n,p} \triangleq \beta_p(nT_s)$ where T_s is the sampling interval at the receiver. Let us assume that the impulse response has finite support, and consider its discretized version

$$\underline{\mathbf{h}}_n^{(i,j)} \triangleq \left[h_{n,0}^{(i,j)}, \dots h_{n,L-1}^{(i,j)}\right]^T, \quad (2)$$

with L chosen such that all the channel coefficients outside the lag interval $[0 \dots (L-1)T_s]$ are zero. Let us further stack these into a row vector with $N_t N_r L$ coefficients

$$\underline{\mathbf{h}}_n \triangleq \left[\underline{\mathbf{h}}_n^{(1,1)^T} \dots \underline{\mathbf{h}}_n^{(1,N_r)^T}, \underline{\mathbf{h}}_n^{(2,1)^T} \dots \underline{\mathbf{h}}_n^{(N_t,N_r)^T} \right]^T. \quad (3)$$

We emphasize the fact that $\underline{\mathbf{h}}_n$ consitutes a snapshot of all the channel impulse response coefficients at time nT_s. With this notation, (1) can be rewritten in more compact form as

$$\underline{\mathbf{h}}_n = \mathbf{G}\underline{\mathbf{b}}_n \quad (4)$$

where $\underline{\mathbf{a}}_p^{(i,j)} \triangleq \alpha_p^{(i,j)} \left(\delta_{l_p^{(i,j)}}(0) \dots \delta_{l_p^{(i,j)}}(L-1) \right)$, $\underline{\mathbf{a}}_p \triangleq \left(\underline{\mathbf{a}}_p^{(1,1)} \dots \underline{\mathbf{a}}_p^{(1,N_r)}, \underline{\mathbf{a}}_p^{(2,1)} \dots \underline{\mathbf{a}}_p^{(N_t,N_r)} \right)^T$, $\mathbf{G} \triangleq [\underline{\mathbf{a}}_1, \dots, \underline{\mathbf{a}}_P]$, and $\underline{\mathbf{b}}_n \triangleq (\beta_{n,1}, \dots, \beta_{n,P})^T$.

III. Spectral factorization and linear estimation

In this section, we outline the possible improvements in channel tracking that can be achieved by deconstructing a specular channel, *i.e.* by separating the time and space properties as enounced in the previous section before doing any kind of smoothing or prediction. We seek to model the discrete-time random process $\{\underline{\mathbf{h}}_n\}$ from its noisy measurements $\underline{\tilde{\mathbf{h}}}_n = \underline{\mathbf{h}}_n + \underline{\mathbf{v}}_n$ where the noise $\{\underline{\mathbf{v}}_n\}$ is white Gaussian, iid, independent from $\{\underline{\mathbf{h}}_n\}$. Assuming that both $\{\underline{\mathbf{h}}_n\}$ and $\{\underline{\mathbf{v}}_n\}$ are wide-sense stationary (WSS), let us define the (matrix) covariances

$$\mathrm{R}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(u) \triangleq \mathcal{E}_n\left[\underline{\tilde{\mathbf{h}}}_{n+u} \underline{\tilde{\mathbf{h}}}_n^H \right] \quad \text{and} \quad \mathrm{R}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(u) \triangleq \mathcal{E}_n\left[\underline{\mathbf{h}}_{n+u} \underline{\tilde{\mathbf{h}}}_n^H \right]$$

where $\mathcal{E}_n[\cdot]$ is the expectation operator taken over n, and the z-transforms

$$\begin{aligned} \mathrm{S}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(z) &\triangleq \sum_{u=-\infty}^{+\infty} \mathrm{R}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(u)\, z^{-u} \quad \text{and} \\ \mathrm{S}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(z) &\triangleq \sum_{u=-\infty}^{+\infty} \mathrm{R}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(u)\, z^{-u}. \end{aligned} \quad (5)$$

The best linear estimator (in terms of mean square error) of $\underline{\mathbf{h}}_{n+\lambda}, \lambda \geq 0$ given $\{\underline{\tilde{\mathbf{h}}}_k\}_{k=-\infty}^n$ is

$$\underline{\hat{\mathbf{h}}}_{n+\lambda} \triangleq \sum_{i=-\infty}^{+\infty} \mathrm{K}_{n-i} \underline{\mathbf{h}}_i \quad (6)$$

where the matrix filter coefficients K_u are determined in the z-transform domain

$$\mathrm{K}(z) \triangleq \sum_{u=-\infty}^{+\infty} \mathrm{K}_u\, z^{-u} \quad (7)$$

by [5]

$$\mathrm{K}(z) = \left\{ z^{\lambda} \mathrm{S}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(z) \mathrm{L}^{-*}(z^{-*}) \right\}_+ \mathrm{R_e}^{-1} \mathrm{L}^{-1}(z), \quad (8)$$

where $\mathrm{L}(z)$ and $\mathrm{R_e}$ come from the spectral factorization

$$\mathrm{S}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(z) = \mathrm{L}(z) \mathrm{R_e} \mathrm{L}^*(z^{-*}). \quad (9)$$

In general, spectral factorization is hard to compute in the case of vector-valued processes, and (8) is not feasible. Note that due to the independence between $\{\underline{\mathbf{v}}_n\}$ and $\{\underline{\mathbf{h}}_n\}$,

$$\mathrm{S}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(z) = \mathrm{S}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(z) \quad \text{and} \quad \mathrm{S}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(z) = \mathrm{S}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(z) + \mathrm{S}_{\underline{\mathbf{v}}\underline{\mathbf{v}}}(z). \quad (10)$$

A. Specular model and spectral factorization

Under the assumption that the channel variations follow the specular model (4), the covariances become

$$\begin{aligned} \mathrm{R}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(u) &= \mathbf{G}\mathcal{E}_n\left[\underline{\mathbf{b}}_{n+u} \underline{\mathbf{b}}_n^H \right] \mathbf{G}^H + \mathcal{E}_n[\underline{\mathbf{v}}_{n+u} \underline{\mathbf{v}}_n^H] \quad (11) \\ &= \mathbf{G}\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(u)\mathbf{G}^H + \mathrm{R}_{\underline{\mathbf{v}}\underline{\mathbf{v}}}(u), \quad (12) \\ \mathrm{R}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(u) &= \mathbf{G}\mathcal{E}_n\left[\underline{\mathbf{b}}_{n+u} \underline{\mathbf{b}}_n^H \right] \mathbf{G}^H = \mathbf{G}\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(u)\mathbf{G}^H. \quad (13) \end{aligned}$$

Note that in eqs. (11) and (13) the factor $\mathbf{G}$ is independent of the lag u. Therefore, the z-transforms can be factored as

$$\begin{aligned} \mathrm{S}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(z) &= \mathbf{G}\mathrm{S}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(z)\mathbf{G}^H + \mathrm{S}_{\underline{\mathbf{v}}\underline{\mathbf{v}}}(z) \quad \text{and} \quad (14) \\ \mathrm{S}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(z) &= \mathbf{G}\mathrm{S}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(z)\mathbf{G}^H. \quad (15) \end{aligned}$$

Let us define

$$s_{bb}^{(p)}(z) \triangleq \sum_{u=-\infty}^{+\infty} \mathcal{E}_n[\beta_{n+u,p}\beta_{n,p}^*]\, z^{-u} \quad \text{for} \quad p = 1 \dots P. \quad (16)$$

Note that $\mathrm{S}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(z)$ is diagonal, since we assume that the $\beta_p(t)$ are independent:

$$\mathrm{S}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(z) = \mathrm{diag}\left(s_{bb}^{(1)}(z), \dots, s_{bb}^{(P)}(z) \right). \quad (17)$$

Note that this structure allows to obtain the spectral factorization of $\mathrm{S}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(z)$ by performing P independent, scalar spectral factorizations of the $s_{bb}^{(p)}(z)$ (equivalently, this means that the random process $\{\underline{\mathbf{h}}_n\}$ can be acurately tracked by tracking P scalar processes).

In the following sections, we will address the following identifiability problem: assuming the knowledge of the spectrum $\mathrm{S}_{\underline{\tilde{\mathbf{h}}}\underline{\tilde{\mathbf{h}}}}(z)$, we show that if the $s_{bb}^{(p)}(z)$ are linearly independent polynomials, it is possible to identify them up to a permutation and a complex scalar coefficient. Fortunately, this is sufficient for our needs, since all possible solutions yield the same predictor $\mathrm{K}(z)$. Then, we discuss an algorithm that achieves this identification.

IV. Identifiability

In this section, we show that if the spectrums $s_{bb}^{(p)}(z)$ are linearly independent polynomials, the spectral factorization (15) is unique up to a permutation and a scalar coefficient applied to the columns of $\mathbf{G}$.

Let us assume that $\mathbf{G}$ has full column rank, and let $(\underline{\mathbf{c}}_1, \dots, \underline{\mathbf{c}}_P)$ be an orthonormal base of the column subspace of $\mathbf{G}$. Let $\mathbf{C} \triangleq [\underline{\mathbf{c}}_1, \dots, \underline{\mathbf{c}}_P]$. Let $\mathbf{R}$ denote the representation of $\mathbf{G}$ in this base, such that $\mathbf{G} = \mathbf{C}\mathbf{R}$. Let us assume that $\mathrm{S}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(z)$ has an alternative factorization

$$\mathrm{S}_{\underline{\mathbf{h}}\underline{\tilde{\mathbf{h}}}}(z) = \mathbf{H}\mathrm{S}_{\underline{\mathbf{b}}'\underline{\mathbf{b}}'}(z)\mathbf{H}^H, \quad (18)$$

and show that $\mathbf{G}$ and $\mathbf{H}$ are identical up to a permutation and a linear scaling of their columns. Using the fact that $\mathbf{C}^H\mathbf{C} = \mathrm{I}_P$, the decomposition in (15) yields

$$\mathrm{S}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(z) = \mathbf{R}^{-1}\mathbf{C}^H\mathrm{S}_{\underline{\mathbf{h}}\tilde{\underline{\mathbf{h}}}}(z)\mathbf{C}\mathbf{R}^{-H}. \tag{19}$$

Hence, using (18) and defining the $P \times P$ matrix $\mathbf{S} \triangleq \mathbf{R}^{-1}\mathbf{C}^H\mathbf{H}$,

$$\mathrm{S}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(z) = \mathrm{S}\mathrm{S}_{\underline{\mathbf{b}}'\underline{\mathbf{b}}'}(z)\mathrm{S}^H. \tag{20}$$

Therefore we need to prove that S is the product of a permutation matrix and a diagonal matrix. Let $\underline{\mathbf{s}}_i, i = 1 \dots P$ denote the columns of $\mathbf{S}$. The diagonal structure of $\mathrm{S}_{\underline{\mathbf{b}}'\underline{\mathbf{b}}'}(z)$ lets us rewrite

$$\mathrm{S}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(z) = \sum_{i=1}^{P} \underline{\mathbf{s}}_i\underline{\mathbf{s}}_i^H s^{(i)}_{\underline{\mathbf{b}}'\underline{\mathbf{b}}'}(z). \tag{21}$$

This implies that each $\underline{\mathbf{s}}_i\underline{\mathbf{s}}_i^H$ is diagonal, otherwise the off-diagonal terms would yield an identically zero linear combination of $s^{(i)}_{\underline{\mathbf{b}}'\underline{\mathbf{b}}'}(z)$'s, which contradicts the linear independence assumption. This implies that each $\underline{\mathbf{s}}_i$ has at most one non-zero coefficient, which is equivalent to saying that S represents the product of a permutation matrix and a diagonal matrix.

V. Practical identification method

The previous discussion has shown that any factorization of the form of (18) is an equally good way of decomposing $\{\underline{\mathbf{h}}_n\}$ into scalar, independent processes. In this section, we present an algorithm to find one of these decompositions.

Let us assume that the noise level is known, or has been estimated, hence $\mathrm{S}_{\underline{\mathbf{h}}}(z)$ is known, or equivalently, $\mathrm{R}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(u)$ is known for $u \in \mathbb{Z}$. The algorithm that we present here provides a matrix $\mathbf{B}$ that decomposes $\mathrm{S}_{\underline{\mathbf{h}}}(z)$ into independent processes, *i.e.* $\mathbf{B}\mathrm{S}_{\underline{\mathbf{h}}}(z)\mathbf{B}^H$ is diagonal. We restrict the problem to the signal subspace, and consider $\mathbf{C}^H\mathrm{R}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(0)\mathbf{C}$. Since it is positive semi-definite, it can be decomposed according to its eigenstructure:

$$\mathbf{C}^H\mathrm{R}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(0)\mathbf{C} = \mathbf{W}\mathbf{D}\mathbf{W}^H, \tag{22}$$

where $\mathbf{W}$ is a $P \times P$ unitary matrix, and $\mathbf{D}$ is diagonal, and contains the (non-negative) eigenvalues. Notice that $(\mathbf{W}\sqrt{\mathbf{D}})(\mathbf{W}\sqrt{\mathbf{D}})^H$ constitutes a Cholesky decomposition of $\mathbf{C}^H\mathrm{R}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(0)\mathbf{C}$. Since $\left(\mathbf{R}\sqrt{\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(0)}\right)\left(\mathbf{R}\sqrt{\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(0)}\right)^H$ is also a Cholesky decomposition of the same matrix, they are unitarily similar [6], *i.e.* there exist an unitary matrix $\mathbf{Q}$ s.t.

$$\left(\mathbf{R}\sqrt{\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(0)}\right)^H = \mathbf{Q}\left(\mathbf{W}\sqrt{\mathbf{D}}\right)^H. \tag{23}$$

Obviously, finding $\mathbf{Q}$ would let us identify $\mathbf{R}$ up to the scalar uncertainties contained in $\sqrt{\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(0)}$. In order to find it, we use the fact that

$$\begin{aligned}\left(\mathbf{W}\sqrt{\mathbf{D}}\right)^{-1}\mathbf{C}^H\mathrm{R}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(u)\mathbf{C}\left(\mathbf{W}\sqrt{\mathbf{D}}\right)^{-H} \\ = \mathbf{Q}^H\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(u)\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(0)^{-1}\mathbf{Q} \quad \forall u \in \mathbb{Z}.\end{aligned} \tag{24}$$

Under our assumptions, there is a unique way (up to a permutation $\mathbf{P}$) of diagonalizing the spectrum matrix, as demonstrated in section IV, hence if $\mathbf{V}$ is a unitary matrix that diagonalizes $\mathbf{V}\sqrt{\mathbf{D}^{-1}}\mathbf{W}^H\mathbf{C}^H\mathrm{R}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(u)\mathbf{C}\mathbf{W}\sqrt{\mathbf{D}^{-1}}\mathbf{V}^H$ for all $u \in \mathbb{Z}$, then $\mathbf{Q} = \mathbf{P}^T\mathbf{V}$. Finding $\mathbf{V}$ is the well-known simultaneous diagonalization problem [7], and can be solved efficiently using an algorithm based on Jacobi angles [8]. It follows that $\{\underline{\mathbf{h}}_n\}$ is transformed into an arbitrary vector process $\{\underline{\mathbf{y}}_n\}$, with

$$\underline{\mathbf{y}}_n \triangleq \mathbf{V}\sqrt{\mathbf{D}^{-1}}\mathbf{W}^H\mathbf{C}^H\underline{\mathbf{h}}_n = \mathbf{P}\sqrt{\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(0)^{-1}}\underline{\mathbf{b}}_n, \tag{25}$$

and $\mathrm{S}_{\underline{\mathbf{y}}}(z)$ is diagonal: $\mathbf{V}\sqrt{\mathbf{D}^{-1}}\mathbf{W}^H\mathbf{C}^H$ is a possible $\mathbf{B}$. The uncertainties outlined in section IV appear clearly in (25): each component of $\{\underline{\mathbf{y}}_n\}$ is normalized to unitary variance, and the permutation $\mathbf{P}$ is unknown.

Obviously, the theoretical identifiability outlined in section IV, is not realistic in practice, since implementation constraints would restrict the knowledge of $\mathrm{R}_{\underline{\mathbf{h}}\underline{\mathbf{h}}}(u)$ to a limited range of u. Also, the requirement of linear independence of the columns of $\mathbf{G}$, as well as the linear independence of the z-spectrums of the time coefficients, are not guaranteed to be fulfilled in real life. However, our simulations show that these limitations do not seem to incur significant problems in practice.

VI. Applications

One of the interest of a specular channel model is its long-term validity. Since it closely follows the physical channel structure, and hence separates the spatial and temporal properties of the channel, estimation or prediction of the time coefficients only (the $\beta_p(t)$) together with the knowledge of $\mathbf{G}$ provides knowledge of the channel state, following from

$$\begin{aligned}\underline{\mathbf{h}}_n &= \mathbf{C}\mathbf{R}\underline{\mathbf{b}}_n && (26)\\ &= \mathbf{C}\mathbf{W}\sqrt{\mathbf{D}}\mathbf{Q}^H\sqrt{\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(0)^{-1}}\underline{\mathbf{b}}_n && (27)\\ &= \mathbf{C}\mathbf{W}\sqrt{\mathbf{D}}\mathbf{Q}^H\mathbf{P}^{-1}\underline{\mathbf{y}}_n && (28)\\ &= \mathbf{C}\mathbf{W}\sqrt{\mathbf{D}}\mathbf{V}^H\underline{\mathbf{y}}_n && (29)\end{aligned}$$

where we used successively (4), (23), (25), and the definition of $\mathbf{Q}$. Note that these matrices need to be estimated first, and in particular the choice of the number of independent components to consider. In practice, this parameter can be chosen as the number of non-noise eigenvalues of $\mathrm{R}_{\tilde{\underline{\mathbf{h}}}\tilde{\underline{\mathbf{h}}}}(0)$. Then, $\mathbf{C}$ can be obtained by orthogonalizing the corresponding set of eigenvectors. Subsequently, the channel estimate can be obtained through

$$\widehat{\underline{\mathbf{h}}}_n = \mathbf{C}\mathbf{W}\sqrt{\mathbf{D}}\mathbf{V}^H\widetilde{\underline{\mathbf{y}}}_n,$$

where $\widetilde{\underline{\mathbf{y}}}_n$ is obtained by any estimation method (for instance smoothing, linear prediction...) from the $\widehat{\underline{\mathbf{y}}}_n$. Since the coefficients in $\underline{\mathbf{y}}_n$ are independent processes, the burden of the estimation method is greatly decreased. Several kinds

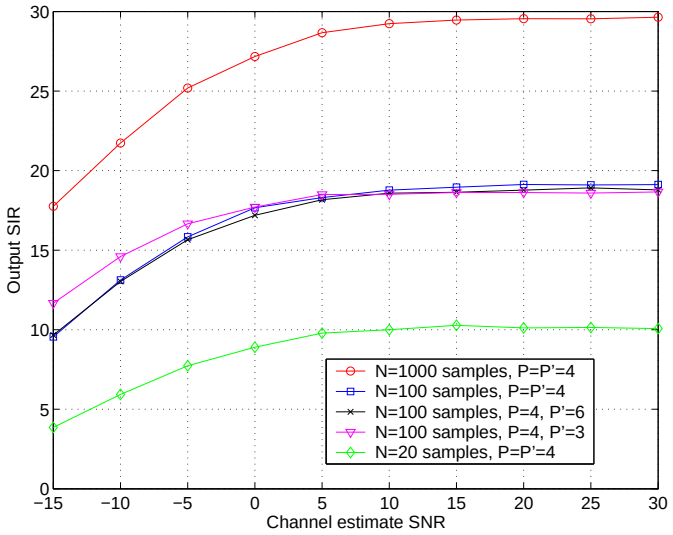

Fig. 1. SIR of the process separation

of processings can be applied at this point, for instance smoothing if the goal is to increase channel estimation acuracy. Prediction can be useful in systems relying on Channel State Information (CSI) at the Transmitter (CSIT) to enhance the link quality: since duplex systems mainly rely on a feedback scheme to transmit the channel state information from the receiver, and since the channel state information can not be fed back in a negligible amount of time, the ability for the transmitter to extrapolate CSI from past values can therefore be an important asset in the actual use of a CSIT-exploiting transmission scheme [9]. The nature of the underlying processes must also be considered. For instance, pure Doppler effect would yield an autoregressive process of order 1.

VII. Simulation results

The algorithm proposed in section V has been proven in section IV to identify perfectly the system in the noiseless case. We present here some results obtained in a more practical simulation setting. We simulated a setting with 2×2 antennas, and a delay spread limited to $L = 5$ samples. The channel is an actual specular channel with $P = 4$ paths, where each path is determined by randomly generated integer lags, uniform DoD and DoAs, a Gaussian gain and a random AR3 process. The algorithm works with approximate covariance matrices estimated from a finite-length measurement interval of N successive realizations of $\{\underline{\mathbf{h}}_n\}$. The number P' of estimated independent random processes to estimate is set artificially, and several cases ($P' > P$, $P' < P$, $P' = P$) are presented here.

The figure of merit used in these simulations is derived from the fact that in the absence of noise, with the notations of section V, $\mathbf{P}^{\mathbf{T}}\mathbf{V}\sqrt{\mathbf{D}^{-1}}\mathbf{W}^H\mathbf{C}^H\mathbf{G} = \sqrt{\mathrm{R}_{\underline{\mathbf{b}}\underline{\mathbf{b}}}(0)^{-1}}$, *i.e.* it is a diagonal matrix. When noise is present, this matrix is computed from the true $\mathbf{G}$ and the specular model as estimated by the proposed algorithm, hence it is not perfectly diagonal. Hence, denoting $[x_{i,j}]_{i,j} \triangleq \mathbf{P}^{\mathbf{T}}\mathbf{V}\sqrt{\mathbf{D}^{-1}}\mathbf{W}^H\mathbf{C}^H\mathbf{G}$, $x_{p,p}$ is the amount of energy from $\{\beta_{n,p}\}_n$ that is correctly attributed to the pth estimated process, and the $x_{i,p}, i \neq p$ represents the crosstalk from other processes. Thus, we define a global signal-to-interference ratio (SIR) as

$$SIR \triangleq \frac{\sum_{p=1}^{P} x_{p,p}^2}{\sum_{p=1}^{P}\sum_{i\neq p} x_{p,i}^2}. \tag{30}$$

This value is plotted on Figure 1 for various configurations, with respect to the SNR of the raw channel estimates $\underline{\tilde{\mathbf{h}}}_n$.

These results clearly show the influcence of the the quality of estimation of the covariances: increasing the number of realizations used for estimating the channel statistics from $N = 20$ to $N = 100$, then 1000, has a relatively bigger influence than the SNR variations over the range pictured here. The influence of overestimating the number of paths can be estimated by comparing the $P' = P = 4$ to the $P' = 6$ case. Overestimation of the number of paths yields an almost negligible decrease in the SIR of the P correctly identified paths. The case where $P' < P$ is interesting in that it represents the resilience of the identification algorithm to model mismatch, *i.e.* when the signal does not conform to the assumptions that support our method. In this case, the SIR criterion is only computed on the P' separated processes, which are statistically the strongest. This explains the fact that we observe a slight SIR increase in this case, for low input SNR values, and goes to prove that the P' strongest paths are correctly identified. However, this metric is hiding the fact that the channel is not fully analyzed, and hence not as predictable. Evidencing this would require a more involved simulation setup, where the modeling error variance would be considered.

VIII. Conclusion

We presented a channel modeling method based on the assumption that the channel follows a specular structure. We showed how this structured model, by separating space and time-components, lends itself to simplified tracking, including smoothing and prediction, once the underlying space and time characteristics are separated. We showed that under mild assumptions on the channel characteristics, these components are identifiable, and proposed a method based on simultaneous diagonalization of the covariance matrices that achieves the identification. We evaluated the performance of the proposed method through simulations.

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